Микроэкономика 2 — ФЭН, 2020 final

ФЭН Микроэкономика 2 2020 final

PROBLEM 1

Short questions — 40 points + 10 bonus points

Questions 1–4 may have multiple correct answers. In Questions 5–7, write only the answer; derivations and proofs are not required.

Question 1 — 5 points

Consider a 2×2 pure-exchange economy with two individuals, A and B, and two goods, 1 and 2.

Their preferences are represented by

$$u_A(x_A^1, x_A^2) = \frac{1}{2}\ln(x_A^1) + \frac{1}{2}\ln(x_A^2),$$

$$u_B(x_B^1, x_B^2) = \sqrt{x_B^1 x_B^2}.$$

Their endowments are

$$e_A = (4, 0), \qquad e_B = (0, 4).$$

Which allocation or allocations are fair?

1. $x_A^1 = 4,\ x_A^2 = 4,\ x_B^1 = 0,\ x_B^2 = 0.$
2. $x_A^1 = 2,\ x_A^2 = 2,\ x_B^1 = 2,\ x_B^2 = 2.$
3. $x_A^1 = 1,\ x_A^2 = 1,\ x_B^1 = 3,\ x_B^2 = 3.$
4. $x_A^1 = 1,\ x_A^2 = 1,\ x_B^1 = 1,\ x_B^2 = 1.$
5. $x_A^1 = 3,\ x_A^2 = 3,\ x_B^1 = 3,\ x_B^2 = 3.$

Question 2 — 5 points

There are N individuals in country X. Let

$$U_i \geq 0$$

be the utility of citizen $i \in \{1, \ldots, N\}$.

Choose every specification that **cannot** be used as a social-welfare function for country X .

1. $$W(U_1, \ldots, U_N) = \frac{\sum_{i=1}^{N} U_i}{U_N}.$$

2. $$W(U_1, \ldots, U_N) = \sqrt{U_1 \cdots U_N}.$$

3. $$W(U_1, \ldots, U_N) = U_N.$$

4. $$W(U_1, \ldots, U_N) = \max\{U_1, \ldots, U_N\}.$$

5. $$W(U_1, \ldots, U_N) = \left(\frac{\sum_{i=1}^{N} U_i}{N} \right)^{-1}.$$

Question 3 — 5 points

Choose every **false** statement.

1. In a first-best contract, a risk-neutral principal must offer a risk-averse agent full insurance in the presence of stochastic output.

2. In a simple Spence signaling model, separating equilibria always exist in which more productive workers invest in education and less productive workers acquire no education.

3. When choosing between $e = 1$ and $e = 0$, it is always optimal for a principal to implement positive effort in the presence of moral hazard.

4. An agent whose preferences over wealth $W \geq 1$ are represented by

   $$u(W) = -\frac{W^{-2} - 1}{2}$$

   always demands insurance.

5. Traffic lights can be treated as a public good.

Question 4 — 5 points

Two roommates, A and B, are considering buying a television.

The television is a public good: it is non-excludable and non-rival in consumption.

Both roommates have reservation value

$$r_i = 80, \qquad i = A, B,$$

and the television costs

$$C = 100.$$

Assume each roommate has enough money to buy the television independently of the other roommate's contribution.

Choose every transfer t from roommate A to roommate B such that it is optimal for B to buy the television and for A not to buy it.

1. $t = 0$.
2. $t = 10$.
3. $t = 30$.
4. $t = 60$.
5. $t = 90$.

Question 5 — 10 points

A risk-neutral firm provides car insurance.

A contract consists of:

- a fee β;
- a reimbursement α, paid in addition to β only if an accident occurs.

A driver owns a car worth

$$W = 400.$$

The driver may be:

- careful, $e = 1$;

- careless, $e = 0$.

The effort cost is

$$c(e) = e.$$

A careful driver has an accident with probability

$$\pi_1 = 0.1,$$

and a careless driver with probability

$$\pi_0 = 0.5.$$

In an accident, the driver loses

$$L = 400.$$

The driver's utility over wealth w is

$$u(w) = \sqrt{w}.$$

Without insurance, the driver chooses the effort level that produces the highest expected payoff.

Find the optimal contract

$$(\alpha^*, \beta^*)$$

under moral hazard and state whether it implements

$$e = 1$$

or

$$e = 0.$$

Question 6 — 10 points

There are two roommates, A and B, and two goods:

- money, m;
- smoke intensity, s.

Preferences are

$$u_A(m_A, s) = m_A + \sqrt{s},$$

$$u_B(m_B, s) = 2m_B - s^2.$$

Each roommate has 2 units of money:

$$e_m^A = e_m^B = 2.$$

Smoke intensity satisfies

$$s \in [0, 1].$$

Roommate A owns the air in the room and may sell roommate B the right to reduce smoke intensity.

Find:

- the equilibrium smoke intensity;
- the market-clearing price ratio

$$\frac{p_m}{p_s}.$$

Question 7 — 10 bonus points

Elsa, the queen of Arendelle, considers building an ice castle in the Nordic mountains.

The castle is a public good. There are four alternatives:

- A_0: no castle;
- A_1: a castle with one high tower;
- A_2: a castle with two high towers;
- A_3: a castle with three high towers.

Three individuals—Anna, Kristoff, and Olaf—represent the citizens of Arendelle. Their net benefits are:

Individual	A_0	A_1	A_2	A_3
Anna	0	270	580	450
Kristoff	0	380	590	950
Olaf	0	130	310	240

Elsa maximizes social net benefit using the VCG mechanism.

Find the taxes paid by:

- Anna;
- Kristoff;
- Olaf.

PROBLEM 2

Externalities — 20 points

Write only the final answers. Proofs and derivations are not required, and supplementary calculations are not graded.

A bear and a hare are roommates. Each operates a perfectly competitive firm and works from home.

The hare knits gloves and sells them at price

$$p_G = 20$$

per pair.

The hare's cost function is

$$c_G(g, x) = g^2 + (6 - x)^2,$$

where:

- g is the hare's output, measured in pairs of gloves;
- x is the amount of time associated with watching *Game of Thrones*.

Watching television raises the hare's productivity up to a point.

The bear produces wooden spoons and sells them at price

$$p_S = 12.$$

The bear dislikes the hare's television watching. The bear's cost function is

$$c_S(s, x) = \frac{s^2}{2} + xs,$$

where s is the bear's output, measured in wooden spoons.

1 (6 points) Find the privately optimal production plan of each firm.

2 (8 points) Compute the socially desirable production plan.

3 (6 points) Mr. Fox gives the bear the property right to silence in the room.

The bear may sell the hare the right to watch *Game of Thrones* at price q per unit of x.

Find q such that the resulting allocation is Pareto efficient.

PROBLEM 3

Adverse selection — 20 points

Perform all necessary derivations.

Consider a used-car market with:

- 100 sellers;
- 100 identical buyers.

Buyers and sellers are risk neutral.

There are:

- q bad cars, called lemons;
- $100 - q$ good cars, called plums.

Buyers value cars at

$$V_P = 1600$$

for a plum and

$$V_L = 800$$

for a lemon.

Sellers' reservation prices are

$$C_P = 1000$$

for a plum and

$$C_L = 600$$

for a lemon.

Each buyer meets exactly one seller.

1 (2 points) Find total surplus in the absence of asymmetric information.

2 (4 points) Assume perfect information.

Find the equilibrium price of lemons and the equilibrium price of plums when buyers and sellers split total surplus equally.

Is the resulting allocation Pareto efficient?

From this point onward, buyers cannot observe the type of car being sold.

3 (2 points) Find the expected value of a car to a buyer when

$$q = 80.$$

4 (4 points) When

$$q = 80,$$

does the market have a pooling or separating equilibrium?

Find the equilibrium and state whether any plums are sold.

As before, buyers and sellers split surplus equally.

5 (2 points) Is the market outcome from part 4 Pareto efficient?

Explain.

6 (6 points) Find the values of q for which a pooling equilibrium exists.

Is such an equilibrium Pareto efficient?

PROBLEM 4

Public goods — 20 points

Perform all necessary derivations and state the optimization problems.

Two families, A and B, live in a village where all houses are made of wood.

To protect their houses from fire, the families invest in a fire station.

Family i has utility

$$u_i(c_i, f) = 2\ln(c_i) + \ln(f), \qquad i \in \{A, B\},$$

where:

- c_i is family i's private consumption expenditure;
- total public-good investment is

$$f = f_A + f_B.$$

Each family has a budget of 100 monetary units and cannot spend more than 100 on c_i and f_i together.

1 (6 points) Find:

- optimal private consumption;
- total investment in fire protection,

when the families choose independently and simultaneously.

2 (6 points) Using a utilitarian social-welfare function, find the Pareto-efficient total investment in fire protection.

Is it greater than, smaller than, or equal to the total investment found in part 1?

Explain the intuition.

3 (8 points) The mayor wants to restore efficiency and increases investment in fire protection by 35 monetary units.

To finance this, the mayor imposes:

- a tax of 20 on family A;

- a tax of 15 on family B.

Both families may still make voluntary contributions to the public good.

Find:

- the new total investment in fire protection;
- family A's voluntary contribution;
- family B's voluntary contribution.

Compare the resulting public-good provision with that in part 1.