Микроэкономика 2 — ФЭН, 2022 final

ФЭН | Микроэкономика 2 | 2022 | final

PROBLEM 1

Matrix Game — 15 points

Consider the following matrix game G:

Player 1 \ Player 2	L	C	R
T	$(10, 10)$	$(3, 13)$	$(5, 1)$
M	$(12, 4)$	$(6, 5)$	$(5, 2)$
B	$(8, 12)$	$(4, 3)$	$(6, 7)$

(a) (3 points) For each player, identify all strictly dominated strategies, if any.

(b) (3 points) Apply the process of Iterative Elimination of Strictly Dominated Strategies (IESDS) to this game to the fullest extent possible. For each player, list all strategies that cannot be eliminated, i.e. strategies that survive IESDS.

(c) (3 points) Find all Nash equilibria in pure strategies, if any.

Now suppose that players play the static game G for two periods. Players observe the outcome of the first period before playing in the second period. Each player's payoff is given by the sum of payoffs from the two periods.

(d) (3 points) Construct a subgame perfect Nash equilibrium (SPNE) in which players play (T, L) in both periods. If this is impossible, explain why such equilibrium cannot exist.

Finally, suppose that players play the static game G for infinitely many periods, and they observe the outcome of the previous play after each period. Each player's payoff is the discounted sum of payoffs from playing G in each period:

$$U_i = \sum_{t=1}^{\infty} \delta^{t-1} u_{it}, \qquad i = 1, 2,$$

where u_{it} is the payoff of player i in period t and $\delta \in (0, 1)$ is a discount factor.

(e) (3 points) Construct a subgame perfect Nash equilibrium (SPNE) in which players play (T, L) in every period. If this is impossible, explain why such equilibrium cannot exist. If possible, derive necessary restrictions on the discount factor δ that are needed for the equilibrium existence.

Hint: Attempt to construct such equilibrium using trigger strategies.

PROBLEM 2

Monopoly Problem — 15 points

There is a single consumer with the following demand function:

$$D(p) = 6 - \frac{p}{4}.$$

On the supply side of the market, there is a single firm. Its total cost of producing quantity q is given by

$$c(q) = \begin{cases} 0, & q = 0, \\ 2q^2 + 2, & q > 0. \end{cases}$$

Suppose that the market is perfectly competitive and the firm acts as a price-taker.

(a) (2 points) Construct the inverse market demand $p(q)$.

(b) (2 points) Construct the firm's supply function $s(p)$.

(c) (3 points) Solve for the perfectly competitive outcome, both quantity and price, and calculate the resulting social welfare.

Now suppose that the firm is a monopoly that is limited to use a tariff function $T(q)$ specified below. For each tariff function, solve for the resulting market outcome and calculate the deadweight loss.

(d) (3 points) Suppose that

$$T(q) = pq,$$

i.e. the firm can pick any p. Solve for the resulting market outcome and calculate the deadweight loss.

(e) (3 points) Suppose that

$$T(q) = pq + \alpha,$$

i.e. the firm can pick any p and α. Solve for the resulting market outcome and calculate the deadweight loss.

(f) (2 points) Suppose that

$$T(q) = pq + \beta q^2 + \alpha,$$

i.e. the firm can pick any p, β and α. Solve for the resulting market outcome and calculate the deadweight loss.

PROBLEM 3

Oligopoly Problem — 20 points

Consider a market with two firms that compete with each other by choosing quantities. The inverse market demand function is given by

$$p(Q) = a - bQ, \qquad a > 0, \quad b > 0,$$

where Q is the total quantity of the good supplied to the market by the two firms:

$$Q = q_1 + q_2.$$

The cost functions of the two firms are given by

$$C_1(q) = c_1 q, \qquad C_2(q) = c_2 q,$$

where

$$0 \leq c_i \leq \frac{a}{2}, \qquad i = 1, 2.$$

(a) (5 points) Solve for a Nash equilibrium of this model.

For parts (b), (c), and (d), assume that

$$a = 15, \qquad b = 1, \qquad c_1 = 0, \qquad c_2 = 3\delta,$$

where $\delta \in [0, 1]$ is a parameter.

(b) (5 points) Solve for a Nash equilibrium of this model and calculate equilibrium profits for both firms.

Suppose that Firm 2 has an option to invest some amount $I \geq 0$ to reduce δ from its current level to zero. The investment decision of Firm 2 is made before firms' decisions on quantities, and this decision is public, i.e. Firm 1 observes it before choosing its quantity.

(c) (5 points) Find all values of I such that Firm 2 would be willing to invest in reducing its marginal costs in the subgame perfect Nash equilibrium (SPNE) of this game.

Additionally, suppose that Firm 1 can offer to pay some amount F to Firm 2 so that Firm 2 **does not invest** I in reducing its own marginal costs.

(d) (5 points) Find all values of I at which such a transaction can be beneficial for both firms in equilibrium.

PROBLEM 4

General Equilibrium with Production — 20 points

Consider a simple Robinson Crusoe economy with two tradable goods, x and y. Both goods require only one input — labor — and the technologies are defined as follows:

$$x(L_x) = \frac{L_x}{a}, \qquad y(L_y) = L_y.$$

The total number of labor units available to Robinson amounts to 100 and is supplied inelastically. Robinson's preferences over goods x and y are given by

$$u(c_x, c_y) = b\sqrt{c_x} + \sqrt{c_y},$$

where c_x indicates the consumption of good x and c_y stands for the consumption of good y . Let the price of good y be normalized to unity:

$$p_y \equiv 1.$$

1 — 2 points

How many markets does the economy have?

2. Competitive equilibrium — 15 points

1. **5 points.** Derive labor demand and supply of goods x and y.

   Hint: Do not forget that you need to define demand and supply for any price vector $(p_x, 1, w)$.

2. **5 points.** Derive demand for goods x and y.

3. **5 points.** Find the market-clearing prices and the equilibrium quantities for **all** goods traded in the economy.

   Hint: Look at labor demand and supply of goods x and y and check whether the market-clearing conditions hold for all the price intervals you have defined.

3 — 3 points

Is the competitive equilibrium from Point 2 Pareto efficient? Explain your answer.

Parameter versions

Version	a	b
A	2	1
B	2	2
C	3	$\sqrt{2}$
D	4	$\sqrt{2}$

PROBLEM 5

Production Externalities — 10 points

Consider two perfectly competitive firms — a honey factory (H) and a flower producer (F). Let

$$p_H = c$$

be the price of honey and

$$p_F = 10$$

the price of flowers.

The F-firm has the following cost structure:

$$c_F(f, p) = \frac{f^2}{2} + (p - d)^2,$$

where f denotes the output of the F-firm and p is associated with the use of a magic powder to make flowers grow faster. However, the powder works magically not only for flowers, but also for bees coming to pollinate them: the quality and quantity of honey they produce improve. In other words, the use of the magic powder by the F-firm reduces the cost of the H-firm:

$$c_H(h, p) = \frac{h^2}{2} + 2h - ph,$$

where h is the output of the H-firm.

1 — 3 points

Find the optimal production plan from each firm's viewpoint.

2 — 3 points

Compute the Pareto-efficient production plan.

3 — 4 points

To restore efficiency, the government decides to introduce a Pigouvian tax: from now on, the F-firm must pay τ per unit of powder used, p. Find the optimal value of τ.

Parameter versions

Version	c	d
A	12	3
B	18	4
C	20	3
D	14	2

PROBLEM 6

Public Goods — 20 points

Consider two families, A and B, who must decide how much to invest in a common non-excludable playground. Let x_A and x_B denote spending by family A and family B, respectively, and let

$$x = x_A + x_B$$

be the total amount invested in the playground.

Family A has m children, and its payoff is

$$\pi_A(x_A, x_B) = m \ln(x_A + x_B) - x_A.$$

There are n children in family B, and its payoff is

$$\pi_B(x_A, x_B) = n \ln(x_A + x_B) - x_B.$$

The families make their investment decisions about x_A and x_B simultaneously. For simplicity, assume neither family faces any budget constraints.

1 — 4 points

Derive the best-response function for family A, namely an optimal investment x_A as a function of x_B.

2 — 4 points

Derive the best-response function for family B.

3 — 6 points

Find the equilibrium investment profile (x_A^*, x_B^*). What is the total amount of money invested in the common playground? Do you observe any free riding? If yes, by which family?

4 — 6 points

Find the Pareto-efficient total investment, namely $x_A + x_B$, in the common playground. Is it greater than, smaller than, or equal to the one observed in Point 3? Explain the intuition.

Parameter versions

Version	m	n
A	2	3

Version	m	n
B	2	4
C	3	4
D	3	5