Микроэкономика 2 — ФЭН, 2023 final

ФЭН | Микроэкономика 2 | 2023 | final

PROBLEM 1

Domestic Monopoly vs. Worldwide — 25 points

Consider a monopolist with the cost function

$$C(q) = q^2 + 10q.$$

The monopolist operates in a domestic market with the demand function

$$D(p) = 80 - 2p.$$

(a) (5 points) Construct the inverse demand $p(q)$ for the domestic market.

(b) (10 points) Solve for the monopoly outcome when it operates in the domestic market and is limited to linear prices: quantity, price, and profit.

(c) (10 points) Now suppose that the monopolist can additionally sell any amount of the good to the world at the world price p^w, while retaining the ability to sell domestically at any desired price. Solve for the monopoly outcome — quantities sold domestically and internationally, domestic price, and aggregate profit — when

$$p^w = 34.$$

PROBLEM 2

Competition in Quantity — 25 points

Consider a market with the following demand function:

$$D(p) = 12 - p.$$

There are N identical firms that compete with each other in quantities. The cost function of a representative firm is given by

$$C(q) = \begin{cases} 0, & q = 0, \\ \frac{3}{2} + 0.5q^2, & q > 0. \end{cases}$$

(a) (8 points) Solve for the symmetric Cournot equilibrium with N firms, assuming that N is small enough for all firms to be active in equilibrium, i.e. find individual quantity and market price as a function of N.

(b) (5 points) Calculate the equilibrium profit of a firm in the Cournot equilibrium of part (a) and establish the maximum number of firms that can be active in equilibrium.

For the rest of the question, assume that there are only two firms, $N = 2$, and the market demand is now given by

$$D(p) = 14 - p.$$

Firms compete with each other according to the Stackelberg model: Firm 1 picks q_1, Firm 2 observes q_1, and then chooses q_2.

(c) (7 points) Derive the best-response function of Firm 2 for any choice of $q_1 \geq 0$. Show that it is never profitable for Firm 1 to drive Firm 2 out of the market, i.e. to pick a sufficiently high q_1 such that Firm 2 optimally chooses $q_2 = 0$.

(d) (5 points) Solve for the Stackelberg equilibrium. Finding equilibrium quantities is sufficient.

PROBLEM 3

General Equilibrium with Production — 25 points

Consider an economy with two households, A and B, and two goods, consumption c and labor L. The households have identical preferences defined over c and L:

$$u_i(c_i, L_i) = a \ln(c_i) + (1 - L_i), \qquad i \in \{A, B\},$$

where

$$l_i = 1 - L_i \in [0, 1]$$

corresponds to leisure, or free time.

Household A owns a firm that produces the consumption good according to

$$Y(L) = b\sqrt{L},$$

where $Y(L)$ is the output of the firm. Household B does not have any non-labor income. Assume all markets are perfectly competitive. Let p be the price of the consumption good and w the wage rate. Normalize

$$p = 1.$$

1 — 10 points

Find a Pareto-efficient allocation for this economy if social welfare is measured as the sum of the households' utilities:

$$SW(\cdot) = u_A(\cdot) + u_B(\cdot).$$

Hint: Keep in mind that a Pareto-efficient allocation does not need to be unique. Also, the households may not want to devote all their time to productive activities: $L_i < 1$ can be optimal for some $i \in \{A, B\}$ or for both.

2 — 12 points

Find competitive equilibrium for this economy. Do both households supply a strictly positive quantity of labor in equilibrium?

Hints:

- *Do not forget that household A solves the utility-maximization problem and the profit-maximization problem separately.*

- *To solve for competitive equilibrium, it is sufficient to consider the labor market only, thanks to Walras' law.*

3 — 3 points

Is the competitive equilibrium from Point 2 Pareto efficient? Explain your answer.

Parameter versions

Version	a	b
1	$\frac{1}{4}$	2
2	$\frac{1}{16}$	4
3	$\frac{1}{9}$	6
4	$\frac{1}{25}$	5

PROBLEM 4

Public Goods — 25 points

Two families, A and B, live in a village where all houses are made of wood. To protect their houses from fires, the families must invest in a fire station. The utility each family derives from private and public consumption is

$$u_i(c_i, f) = k\ln(c_i) + \ln(f), \qquad i \in \{A, B\},$$

where c_i corresponds to family i's expenditures on private consumption and

$$f = f_A + f_B$$

is the total public-good investment.

Families A and B face identical budget constraints and cannot spend on c_i and f_i more than d monetary units each:

$$c_i + f_i \leq d, \qquad i \in \{A, B\}.$$

1 — 8 points

Find the optimal private consumption and total investment in fire protection if the families make their decisions independently and simultaneously.

2 — 8 points

Find the Pareto-efficient total investment in fire protection if social welfare is measured as the sum of the families' utilities:

$$SW(\cdot) = u_A(\cdot) + u_B(\cdot).$$

Is it greater than, smaller than, or equal to the one observed in Point 1? Explain the intuition.

3 — 9 points

The mayor of the village wants to restore efficiency and decides to increase investment in fire protection by 40 monetary units. To do so, the mayor imposes a tax of 20 monetary units on each family. At the same time, both families can still make a voluntary contribution to the public good.

Find the new total investment in fire protection. What is the voluntary contribution of family A? Family B? How does this compare to the level of public-good provision from Point 1?

Parameter versions

Version	k	d
1	2	200
2	2	160
3	3	210
4	3	280