Микроэкономика 2 — ФЭН, 2024 final

ФЭН | Микроэкономика 2 | 2024 | final

PROBLEM 1

Monopoly Problem — 30 points

A monopoly firm operates in a market with two consumers. The inverse demand functions of these consumers are given by

$$p_1(q) = 10 - \frac{q}{2}, \qquad p_2(q) = 8 - \frac{q}{2}.$$

The total cost function of the monopoly is

$$C(q) = 5q.$$

(a) (6 points) Construct the market demand $D(p)$ and the inverse market demand $p(q)$.

(b) (4 points) Construct the marginal-revenue function for the monopoly.

(c) (5 points) Solve for the monopoly outcome when it is limited to linear prices: calculate quantity, price, and profit.

For the rest of the question, suppose that the monopoly is allowed to use a single two-part tariff

$$T(q) = F + pq,$$

where $F \geq 0$ is a fixed fee and $p \geq 0$ is a per-unit price.

(d) (5 points) Suppose that the monopoly cannot serve both consumers, i.e. it has to choose whether to serve Consumer 1 or Consumer 2. Solve for the monopoly outcome: parameters F and p of the optimal tariff, quantity, and profit.

(e) (8 points) Now suppose that the monopoly can serve both consumers. Solve for the monopoly outcome assuming that both consumers are served: parameters of the optimal tariff, quantity, and profit.

(f) (2 points) Using your results from parts (d) and (e), derive the best two-part tariff for the monopoly when it can serve both markets but does not have to serve both of them.

PROBLEM 2

Public Goods — 40 points

Consider two families, A and B, who must decide how much to invest in a common non-excludable playground. Let x_A and x_B denote investment by family A and family B, respectively, and let

$$x = x_A + x_B$$

be the total amount invested in the playground.

The payoff of family A is

$$\pi_A(x_A, x_B) = m\ln(x_A + x_B) - x_A, \qquad m > 0,$$

and the payoff of family B is

$$\pi_B(x_A, x_B) = 2\ln(x_A + x_B) - x_B.$$

For simplicity, suppose that the families do not face any budget constraints.

For now, assume that

$$m = 4.$$

(a) (5 points) How much will family A invest in the common playground if family B contributes $x_B = 1$?

(b) (5 points) Suppose the families make their investment decisions simultaneously. Solve for the Nash equilibrium of this game and find the total investment in the common playground, namely $x_A + x_B$.

(c) (10 points) Next, suppose the families make their investment decisions sequentially. Specifically, family A chooses x_A first, then family B observes x_A and chooses x_B. Solve for the SPNE of this game and find the total investment in the common playground. Is it greater than, smaller than, or equal to the one observed in part (b)?

(d) (5 points) Find the Pareto-efficient total investment in the common playground. Is it greater than, smaller than, or equal to what you found in parts (b) and (c)? Explain the intuition.

Further, assume that m can take any strictly positive value.

(e) (5 points) Suppose the families make their investment decisions simultaneously. Show that for $m \neq 2$, there does not exist a Nash equilibrium in which both families invest a strictly positive amount in the common playground.

(f) (10 points) Assume $m > 2$ and consider the case in which the families make their investment decisions sequentially: family A moves first and family B follows after observing family A's investment. Find all values of m, consistent with $m > 2$, for which family A chooses $x_A = 0$ in the SPNE.

PROBLEM 3

General Equilibrium with Production — 30 points

Consider an economy with two households, A and B, and three goods — coconuts (c), fish (f), and labor (L). The households have identical preferences defined over c, f, and L:

$$u_i(x_i^c, x_i^f, L_i) = 2\sqrt{x_i^c} + \sqrt{x_i^f} + L_i, \qquad i \in \{A, B\},$$

where x_i^j corresponds to the quantity of good $j \in \{c, f\}$ in the consumption bundle of household $i \in \{A, B\}$ and L_i indicates the amount of labor household i uses.

Household A owns a farm where she produces coconuts, and a fishery belongs to household B. The production of both consumption goods requires only one input — labor — and the technologies are

$$c(L_c) = 4L_c, \qquad f(L_f) = 4L_f,$$

where L_j is the amount of labor used by firm $j \in \{c, f\}$.

Household A holds 50 labor units, which can be used completely or only partially, and household B holds 40 labor units.

Assume all markets are perfectly competitive. Let p_j be the price of good $j \in \{c, f\}$ and let w indicate the wage rate. Normalize the price of coconuts to unity:

$$p_c = 1.$$

(a) (10 points) Find the production possibilities frontier (PPF) for this economy. Express c as a function of f and compute the marginal rate of transformation.

(b) (15 points) Find competitive equilibrium for this economy.

Hints:

- *Do not forget that each household solves the utility-maximization problem and the profit-maximization problem separately.*
- *To solve for competitive equilibrium, it is sufficient to consider the market-clearing conditions for two markets only, thanks to Walras' law.*
- *With two households, focus on aggregate labor supply and aggregate demand for the consumption goods.*

- *Take a closer look at the households' preferences and check whether* $x_i^c = 0$ *or* $x_i^f = 0$ *can hold in the optimum. This will help you solve for the equilibrium price vector.*

(c) (5 points) Is the competitive equilibrium from part (b) Pareto efficient? Explain your answer.