Математический анализ 1 — МИЭФ, 2022 midterm

МИЭФ | Математический анализ 1 | 2022 | midterm

Рисунки пока рендерятся в тестовом режиме и могут отличаться от исходных материалов.

QUESTION 1

On the first grid, sketch a graph of a function f such that:

(a) $f(x)$ is odd.

(b) $y = f(x)$ has asymptote $y = x - 1$ as $x \to +\infty$.

(c) $y = f(x)$ has only two discontinuities, both of jump type.

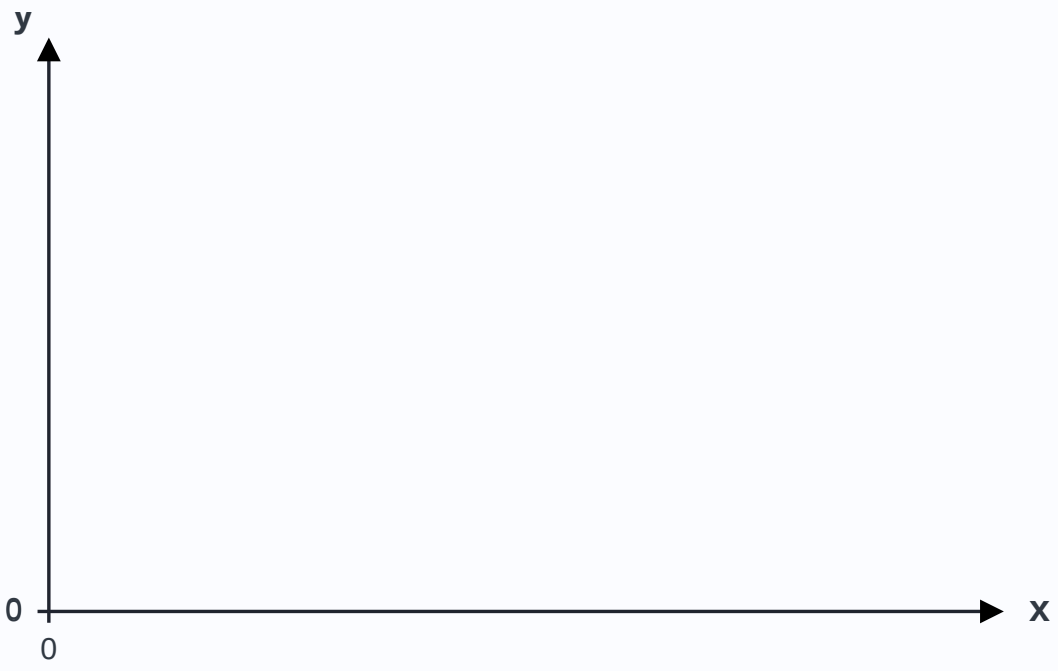
(d) $\lim_{x \to 1+} f(x) = 1$ and $\lim_{x \to -1+} f(x) = -3$.

(e) $f(0) = f(1)$.

Then on the second grid sketch

$$y = |f(|x|) - 2|,$$

and label discontinuities.



Two grids for f and |f(|x|)-2|

QUESTION 2

Consider

$$f(x) = \begin{cases} \dfrac{\sqrt{1-\sin(\alpha x^2)} - \cos x}{x \arctan x}, & x < 0, \\ \beta, & 0 \le x \le 8, \\ \dfrac{7 - \sqrt{6x+1}}{x^2 - 13x + 40}, & x > 8. \end{cases}$$

(a) Find $\lim_{x \to 8+} f(x)$.

(b) Find $\lim_{x \to 0-} f(x)$. Use the standard equivalences for $\arctan u$, $\sin u$, $\sqrt{1+u} - 1$, and $1 -$
$cosu$ as $u \to 0$.

(c) What values of α and β, if any, make f continuous on $\mathbb{R}$? Explain.

(d) Find $\lim_{x \to -\infty} f(x)$. Justify.

QUESTION 3

Consider

$$f(x) = \frac{\ln|\ln x|}{x - e}.$$

(a) Find the domain D.

(b) Give all vertical asymptotes of $y = f(x)$, if any. Explain.

(c) Does $y = f(x)$ have a horizontal asymptote? Justify.

(d) Find $p \notin D$ such that f is defined on both sides of p and $\lim_{x \to p-} f(x) = \lim_{x \to p+} f(x) = L$. Use $\ln(1+u) \sim u$ to find L.

QUESTION 4

Consider the sequence in which each $1/n$ appears n times:

$$1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{4}, \ldots$$

(a) Find a_{2021}.

(b) Find $\lim_{n\to\infty} 2021^{a_n}$.

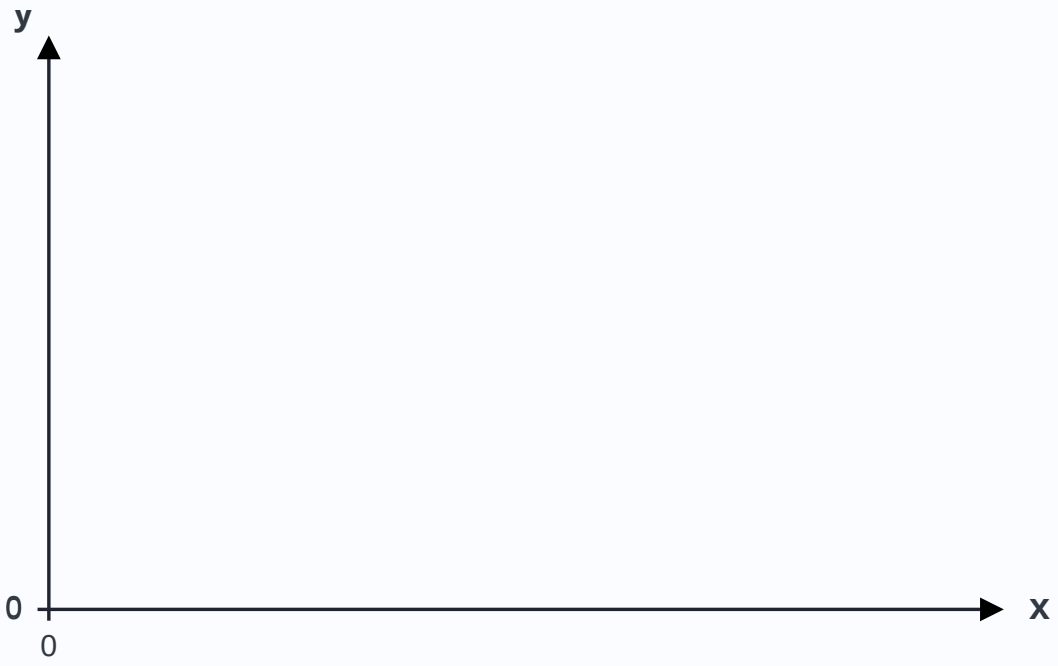
(c) Find $\lim_{n\to\infty} a_n\sqrt{n}$.

QUESTION 5

The graphs of functions f (solid line) and g (dashed line) are shown. At $x = p$, function f has a removable discontinuity, while g has a jump discontinuity. Moreover,

$$f(p) = g(p) = p.$$

Which statement is false?



Graphs of f and g near x=p

Нужно проверить: "The figure is schematic; exact coordinates are not relevant."

1. $\lim_{x \to p} f(x)g(x)$ exists.
2. $\lim_{x \to p}(f(x) - g(x))$ exists.
3. $\lim_{x \to p}(f(x) + g(x))$ does not exist.
4. $\lim_{x \to 0} \frac{f(x)}{g(x)}$ does not exist.

QUESTION 6

Find

$$\lim_{x\to-\infty}\frac{(3x-1)(2-4x)}{|1+3x-6x^2|}.$$

1. -2.
2. -1.
3. 1.
4. 2.

QUESTION 7

The domain of

$$g(x)=\sqrt{\frac{1}{|x+1|}-1}$$

is:

1. $(-\infty,-1)\cup(-1,0]$.
2. $[-2,0]$.
3. $[-2,-1)\cup(-1,0]$.
4. $[-2,+\infty)$.

QUESTION 8

The smallest period of

$$f(x) = \sin(12x) + \cos(18x)$$

is:

1. $\frac{\pi}{18}$.
2. $\frac{\pi}{9}$.
3. $\frac{\pi}{6}$.
4. $\frac{\pi}{3}$.

QUESTION 9

Functions f and g are defined for all x. Function f is odd, and g is even. Which functions must be odd?

I. $f(g(x))$.

II. $g(f(x))$.

III. $f(x)g(x)$.

1. I only.
2. II only.
3. III only.
4. I and III only.

QUESTION 10

Find

$$\lim_{x \to +\infty} \left(\frac{x + 2310}{x - 2310} \right)^{x+2310}.$$

1. e^{1155}.
2. e^{2310}.
3. e^{4620}.
4. $+\infty$.

QUESTION 11

Find

$$\lim_{x \to 0} \frac{5x^3 - 2x^5}{3x^5 - 20x^3}.$$

1. $-\frac{1}{4}$.
2. $-\frac{2}{3}$.
3. $-\frac{1}{10}$.
4. $\frac{5}{3}$.

QUESTION 12

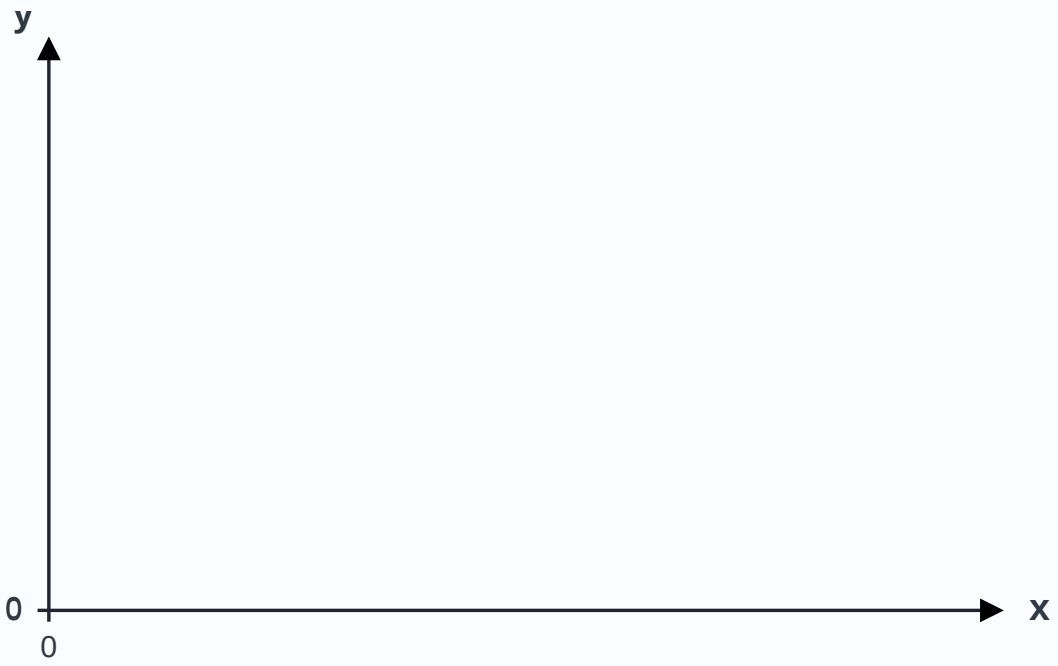
The line $y = 23$ is a horizontal asymptote for which function?

1. $y = \frac{\sin(46x)}{2x}$.
2. $y = \frac{1}{x-23}$.
3. $y = \frac{23x}{1-x}$.
4. $y = \frac{46x^2-x}{1+2x^2}$.

QUESTION 13

Which graph could represent

$$y = f(|2x - 1| - 1), \qquad f(x) = \frac{1}{x}?$$



Four candidate graphs for y = 1/(|2x-1|-1)

Нужно проверить: "Exact candidate shapes should be checked in the source image."

1. a.
2. b.
3. c.
4. d.

QUESTION 14

Compute

$$\lim_{n\to\infty} \frac{24n^3+18n^2-14n+1}{363636n-242424n^2+5-6n^3}.$$

1. $-\frac{1}{10101}$.
2. $\frac{2}{30303}$.
3. 3.
4. -4.

QUESTION 15

Find

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}.$$

1. $\frac{1}{4}$.
2. $\frac{1}{2}$.
3. 2.
4. ∞.

QUESTION 16

Which value of σ makes ϕ continuous on $(-\infty, \infty)$?

$$\phi(x) = \begin{cases} \sigma x^2 + 5x, & x < 2, \\ \dfrac{22}{\sigma - x}, & x \geq 2. \end{cases}$$

1. 3.
2. 1.
3. -1.
4. -3.
5. None of them.

QUESTION 17

Which sequences are convergent?

I. $\left\{\frac{\ln n}{\ln(2n)}\right\}$.

II. $\left\{\frac{(\ln n)^2}{n}\right\}$.

III. $\left\{\frac{\ln n}{n^n}\right\}$.

1. II and III only.
2. I and III only.
3. I and II only.
4. I, II and III.

QUESTION 18

If $9^x - 8^x$ is equivalent to cx as $x \to 0$, then $c =$

1. $3 \ln 2 - 2 \ln 3$.
2. $2 \ln 3 - 3 \ln 2$.
3. $\ln(3/2)$.
4. $\ln(2/3)$.

QUESTION 19

Find

$$\lim_{x \to -5} \frac{6 - \sqrt{41 + x}}{x + 5}.$$

1. -1.
2. $-\frac{1}{6}$.
3. $-\frac{1}{12}$.
4. The limit does not exist.

QUESTION 20

If $f(x)$ is continuous for all real numbers, which statements are true?

I. $f\left(\lim_{x\to x_0} x\right) = \lim_{x\to x_0} f(x)$.

II. $\lim_{n\to\infty} f(x_n) = f(L)$ for any sequence $\{x_n\}$ with $\lim x_n = L$.

III. If $f(a) > 0$, then there exists $\delta > 0$ such that $f(x) > 0$ for $x \in (a-\delta, a+\delta)$.

1. I, II and III.
2. I only.
3. I and II only.
4. I and III only.

QUESTION 21

If f is continuous for all x such that $|x - 23| \leq 10$, which statements must be true?

I. If $f(13)f(33) < 0$, then there exists $\xi \in (13, 33)$ such that $f(\xi) = 0$.

II. There exists $\zeta \in [13, 33]$ such that $f(\zeta) = \max_{13\leq x\leq 33} f(x)$.

III. f attains all values between $f(13)$ and $f(33)$.

1. I and II only.
2. II and III only.
3. I and III only.
4. I, II and III.

QUESTION 22

A sequence starts with $a_1 = 1$ and satisfies

$$a_n = \frac{a_{n-1}}{2} + \frac{1}{a_{n-1}}$$

for $n > 1$. Given that the sequence converges to L, which is true?

1. $L = \frac{1}{\sqrt{2}}$.
2. $L = 1$.
3. $L = \sqrt{2}$.
4. $L = 2$.

QUESTION 23

Find

$$\lim_{x \to +\infty} \frac{\sqrt{x} + 2\sqrt[3]{x} + 5\sqrt[6]{x}}{\sqrt{4x} + \sqrt[3]{1 + 3x}}.$$

1. $\frac{1}{2}$.
2. $\frac{1}{3}$.
3. $\frac{2}{3}$.
4. 1.

QUESTION 24

The sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ satisfy $x_n < y_n < z_n$. Which statements are false?

I. If $\{x_n\}$ and $\{z_n\}$ are convergent, then $\{y_n\}$ is also convergent.

II. If $\lim z_n = \lim x_n$, then $\lim y_n$ exists.

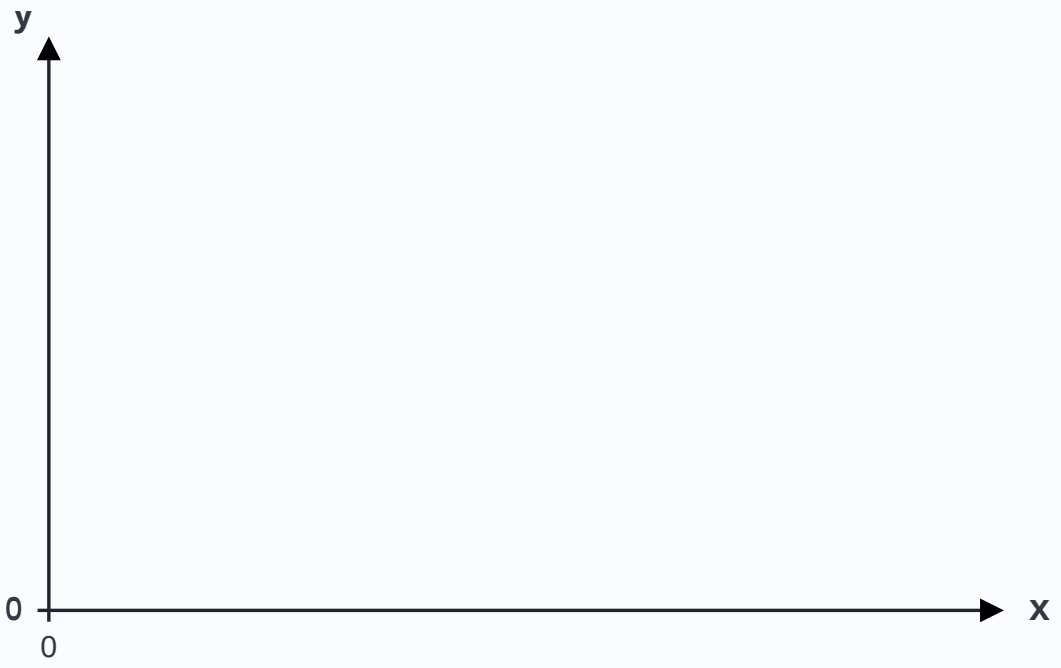
III. If all three sequences converge, then $\lim x_n < \lim y_n < \lim z_n$.

1. I only.
2. I and II only.
3. III only.
4. I and III only.

QUESTION 25

Function f is defined for $x \neq \pm 1$. The graph $y = f(x)$ has two vertical asymptotes $x = -1$ and $x = 1$ and two horizontal asymptotes $y = -1$ and $y = 1$.

Which limit exists?



Graph with vertical asymptotes x=-1 and x=1

Нужно проверить: "The source image should be checked for the direction of divergence at each asymptote."

1. $\lim_{x \to -1} \arctan f(x)$.
2. $\lim_{x \to 1} \arctan f(x)$.
3. $\lim_{x \to -1} \sin f(x)$.
4. $\lim_{x \to 1} \sin f(x)$.

QUESTION 26

Find the asymptote for

$$f(x) = \sqrt{121x^2 + 33x}$$

as $x \to -\infty$.

1. $2y = 22x + 3$.
2. $2y + 22x = -3$.
3. $2y = 22x - 3$.
4. $y = -11x - 3$.

QUESTION 27

Function $f(x)$ is defined for all values of x and

$$\lim_{x \to 0} xf(x) = 1.$$

Which statement can be true?

1. $\lim_{x \to 0-} f(x) = 10$ and $\lim_{x \to 0+} f(x) = 23$.
2. f is continuous at $x = 0$.
3. $\lim_{x \to 0-} f(x) = +\infty$ and $\lim_{x \to 0+} f(x) = -\infty$.
4. $\lim_{x \to 0-} f(x) = -\infty$ and $\lim_{x \to 0+} f(x) = +\infty$.

QUESTION 28

Find

$$\lim_{x \to 2} \left(\frac{x^2 - 2x - 3}{2x^2 - 3x + 1} \right)^{x^2+1}.$$

1. 0.
2. -1.
3. ∞.
4. $\frac{1}{e}$.
5. The expression is not defined in a neighbourhood of $x = 2$.