

# Математический анализ 1 — МИЭФ, 2023 midterm

МИЭФ

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2023

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Рисунки пока рендерятся в тестовом режиме и могут отличаться от исходных материалов.

## QUESTION 1

Let  $f(x) = \sqrt{x}$ .

- (a) Is  $f$  even, odd, or neither? Justify.
- (b) Find the domain of  $f$ . Justify.
- (c) Find the range of  $f$ . Justify.
- (d) Find all points where  $f$  is continuous. Justify.
- (e) Find  $f'(x)$ .

## QUESTION 2

Let  $f(x) = \frac{1}{x^2}$ .

- (a) For what  $x$  is  $f$  discontinuous?
- (b) At each discontinuity  $x_0$ , find  $\lim_{x \rightarrow x_0} f(x)$  or justify non-existence.
- (c) Find all slant or vertical asymptotes. Justify.
- (d) A rational function  $\frac{p(x)}{q(x)}$  satisfies  $\lim_{x \rightarrow \infty} \frac{p(x)}{q(x)} = L$  for all  $x$ . Find  $L$ .

### QUESTION 3

Consider  $f(x) = \frac{1}{x^2}$ .

(a) Classify  $f(x)$  as increasing, nondecreasing, nonincreasing, decreasing, or neither. Justify.

(b) Is  $f(x)$  bounded? Justify.

(c) What conclusion follows from (a) and (b)? Name all theorems used.

(d) If  $f(x) = \frac{1}{x^2}$ , find  $\lim_{x \rightarrow \infty} f(x)$ .

(e) Calculate  $\lim_{x \rightarrow \infty} f(x)$  for  $f(x) = \frac{1}{x^2}$ . Are (d) and (e) consistent?

### QUESTION 4

Let  $f$  be bounded and continuous on  $[a, b]$  and  $f'(x) \geq 0$  for all  $x \in [a, b]$ . Which statements must be true?

(a) There exists  $c \in [a, b]$  such that  $f'(c) = 0$  for all  $c \in [a, b]$ .

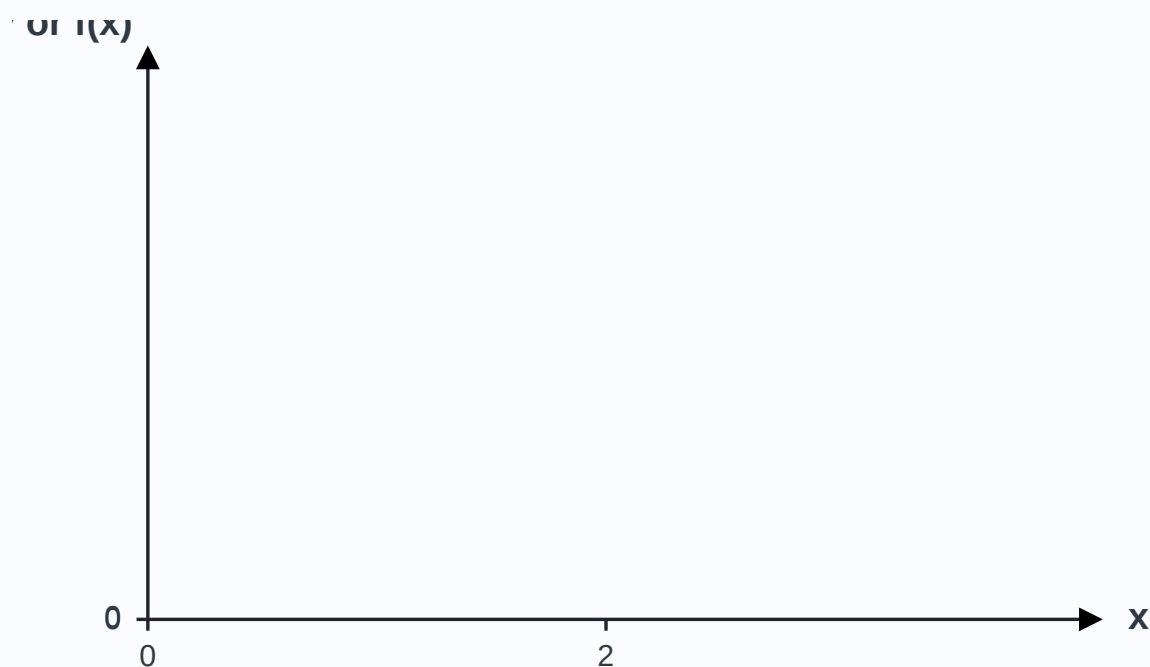
(b)  $f$  does not change sign on  $[a, b]$ .

(c)  $f$  is constant for some finite  $c \in [a, b]$ .

If a statement is always true, explain why and mention relevant theorems. If false, provide a counterexample.

### QUESTION 5

The graph of  $f(x)$  is shown below. Which of the alternatives **a–e** represents the graph of  $f(|x|)$ ?



Source graph of  $f$  and five candidate graphs for  $f(|x|)$

Нужно проверить: "Coordinates are approximate because the source graph provides only tick marks, not an explicit formula."

1. **a**
2. **b**
3. **c**
4. **d**
5. **e**

### QUESTION 6

Let

\_\_\_\_\_

The complete set of solutions of

is:

1.  $-.$

2.  $.$

3.  $.$

4.  $.$

5.  $-.$

### QUESTION 7

Let

Then the inverse function is:

1.

$$\sqrt{\quad}$$

2.

$$\sqrt{\quad}$$

3.

$$\sqrt{\quad}$$

4.

$$\sqrt{\quad}$$

5.

$$\sqrt{\quad}$$

### QUESTION 8

Find the principal period of

—

1. .

2. .

3. .

4. .

5. .

### QUESTION 9

Let

What is the range of ?

1. .

2. .

3. .

4. .

5. .

### QUESTION 10

Let

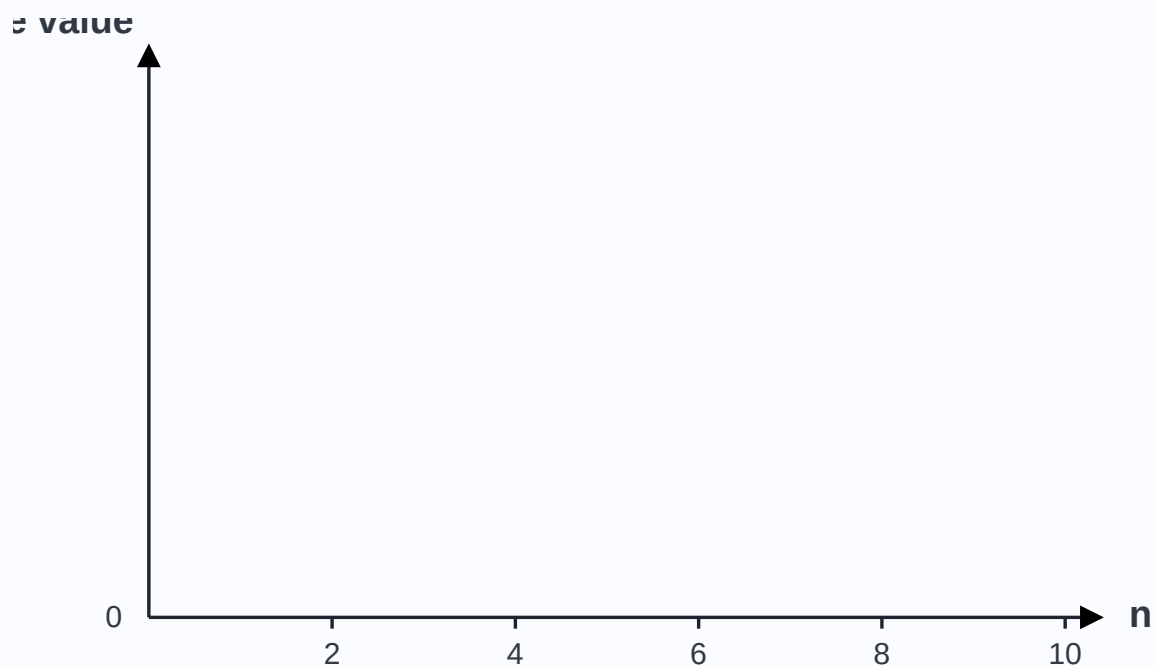
$$f(x) = |x - 1| - |x + 2|$$

The maximum value attained by  $f$  is:

1. at most  $-1$ ;
2.  $-1$ ;
3.  $1$ ;
4.  $2$ ;
5. greater than  $2$ .

### QUESTION 11

Match the five graphs with the five sequences.



Five discrete sequence graphs

Нужно проверить: "The plotted values are qualitative; exact numerical coordinates are not supplied."

Choose the correct matching:

- |    |       |       |                      |       |                      |
|----|-------|-------|----------------------|-------|----------------------|
| 1. | _____ | _____ | _____                | _____ | _____ $\sqrt{\quad}$ |
| 2. | _____ | _____ | _____                | _____ | _____ $\sqrt{\quad}$ |
| 3. | _____ | _____ | _____ $\sqrt{\quad}$ | _____ | _____                |

4.                      \_\_\_\_\_                       $\sqrt{\quad}$                       \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_

5.                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_                       $\sqrt{\quad}$

### QUESTION 12

If

—

for every integer \_\_\_\_\_, then \_\_\_\_\_ equals:

1.                      —.

2.                      —.

3.                      .

4.                      —.

5.                      —.

### QUESTION 13

Which of the following sequences converge?

\_\_\_\_\_

1. I only.
2. II only.
3. I and II only.
4. I and III only.
5. I, II, and III.

### QUESTION 14

Which statements are true?

- I. All bounded sequences are convergent.
- II. If a sequence is unbounded, then it is infinitely large.
- III. An unbounded sequence may have two limits.

1. I only.
2. II only.
3. II and III only.
4. I and III only.
5. None of the statements is true.

### QUESTION 15

Find

\_\_\_\_\_

1. \_\_\_\_\_.
2. \_\_\_\_\_.
3. \_\_\_\_\_.
4. \_\_\_\_\_.
5. \_\_\_\_\_.

### QUESTION 16

Let sequences \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ be such that both difference sequences

are infinitesimally small.

Which of the following sequences must be convergent?

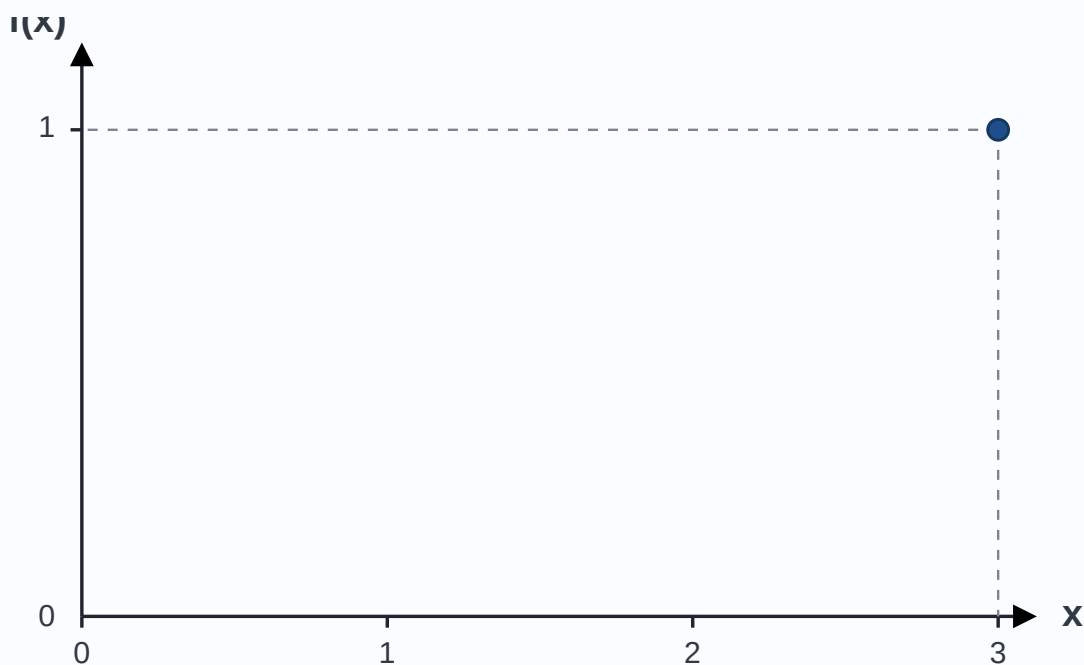
\_\_\_\_\_

1. I only.
2. II only.
3. I and II only.
4. I and III only.
5. I, II, and III.

### QUESTION 17

The graph of a function is shown below.

For which values of does



Piecewise graph used to evaluate two-sided limits

Нужно проверить: "The exact height of the horizontal ray for  $x > 3$  is not labelled; only that it is below 1 matters."

1. only.
2. and only.
3. and only.
4. and only.
5. , , and .

### QUESTION 18

Find

\_\_\_\_\_

1. The limit does not exist.
2.  $\frac{1}{2}$ .
3.  $\frac{1}{3}$ .
4.  $\frac{1}{4}$ .
5.  $\frac{1}{5}$ .

### QUESTION 19

Among the following choices of  $\delta$ , which is the largest that can be used successfully for arbitrary  $\epsilon$  in an  $\epsilon$ - $\delta$  proof of

1.  $\frac{1}{2}$ .
2.  $\frac{1}{3}$ .
3.  $\frac{1}{4}$ .
4.  $\frac{1}{5}$ .
5.  $\frac{1}{6}$ .

### QUESTION 20

Let

\_\_\_\_\_

Which statements are true?

- I. The graph of  $f(x) = \frac{1}{x}$  has a horizontal asymptote  $y = 0$ .
- II. The graph of  $f(x) = \frac{1}{x}$  has a horizontal asymptote  $x = 0$ .
- III. The graph of  $f(x) = \frac{1}{x}$  has a vertical asymptote at  $x = 0$ .

1. I only.
2. II only.
3. III only.
4. I and III only.
5. II and III only.

### QUESTION 21

Find the asymptote of

$$\frac{1}{\sqrt{x}}$$

as  $x \rightarrow \infty$ .

1.  $y = 0$ .
2.  $y = 1$ .
3.  $x = 0$ .
4.  $x = 1$ .
5.  $y = 1$ .

### QUESTION 22

Let

$$f(x) = \frac{1}{x^2 - 1}$$

The graph of  $f(x)$  has:

1. two vertical asymptotes, one horizontal asymptote, and one slant asymptote with nonzero slope;
2. two vertical asymptotes and two slant asymptotes with slopes of different signs;
3. one vertical asymptote and two slant asymptotes with positive slopes;
4. one vertical asymptote and three slant asymptotes with positive slopes;
5. three different asymptotes.

### QUESTION 23

Find

$$\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$$

1.  $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$

2.  $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$

3.  $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}}$

4.  $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}}$

5.  $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}}$

### QUESTION 24

Let  $f$  be continuous at  $a$ , and

$$f(x) = \frac{\sqrt{x} - \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

Then:

1.  $f(a) = \frac{\sqrt{a} - \sqrt{a}}{\sqrt{a} + \sqrt{a}}$

2.  $f(a) = \frac{\sqrt{a} + \sqrt{a}}{\sqrt{a} - \sqrt{a}}$

3.  $f(a) = \frac{\sqrt{a} - \sqrt{a}}{\sqrt{a} - \sqrt{a}}$

4.  $f(a) = \frac{\sqrt{a} + \sqrt{a}}{\sqrt{a} + \sqrt{a}}$

5.  $f(a) = \frac{\sqrt{a} - \sqrt{a}}{\sqrt{a} - \sqrt{a}}$

### QUESTION 25

Let  $f$  be a continuous function defined by

$$f(x) = \sqrt{x - 1}$$

Which interval could be the domain of  $f$ ?

1.  $[-1, 1]$
2.  $[-1, 0]$
3.  $[0, 1]$
4.  $[1, 2]$
5.  $[-1, 2]$

### QUESTION 26

Let  $f$  be continuous on the closed interval  $[a, b]$ . If

then the Intermediate Value Theorem guarantees that:

1.  $f$  is constant; ;
2.  $f$  is constant for some  $c$  between  $a$  and  $b$ ; ;
3.  $f$  is constant for every  $c$  between  $a$  and  $b$ ; ;
4.  $f$  is constant for at least one  $c$  between  $a$  and  $b$ ; ;
5.  $f$  is constant for at least one  $c$  between  $a$  and  $b$ . ;

### QUESTION 27

According to the Intermediate Value Theorem, which statements are true?

I. The equation

has at least one root in  $[-1, 1]$ .

II. The function

takes the value  $0$  in  $[-1, 1]$ .

III. The function

takes the value  $1$  in  $[-1, 1]$ .

1. III only.
2. I and III only.
3. II and III only.
4. I and II only.
5. I, II, and III.

### QUESTION 28

Let  $f(x) = x^3 + 2x^2 + 3x + 4$ . The polynomial equation

must have:

1. only one root;
2. at least one root;
3. an even number of roots;
4. no negative roots;
5. no positive roots.

### QUESTION 29

At how many points do the graphs

intersect?

1. None.
2. One.
3. Two.
4. Three.
5. Four.

**QUESTION 30**

Let  $n$  be the number of real solutions of

in  $(-1, 1)$ , and let  $m$  be the number of real solutions outside  $(-1, 1)$ .

Which statement is true?

1.  $n = 0$  and  $m = 2$ .
2.  $n = 1$  and  $m = 1$ .
3.  $n = 2$  and  $m = 0$ .
4.  $n = 1$  and  $m = 2$ .
5.  $n = 2$  and  $m = 1$ .

### QUESTION 31

Let

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Which statement is true?

1.  $f$  is continuous for every  $x$ .
2.  $f$  has a removable discontinuity at  $x = 0$ .
3.  $f$  has a jump discontinuity at  $x = 0$ , and  $f$  is continuous for every  $x > 0$ .
4.  $f$  has a jump discontinuity at  $x = 0$ , and  $f$  is continuous for every  $x < 0$ .
5.  $f$  has a Type II discontinuity at  $x = 0$ .

### QUESTION 32

Consider

,

,

The function has:

1. only one jump discontinuity;
2. two jump discontinuities;
3. one Type II discontinuity only;
4. one jump discontinuity and one Type II discontinuity;
5. one jump discontinuity and one removable discontinuity.

### QUESTION 33

Consider

$$f(x) = \frac{1}{x^2} \ln \left( \frac{x^2 + 1}{x^2 - 1} \right)$$

Let  $N$  be the number of Type II discontinuities of  $f$ , and let  $M$  be the number of distinct asymptotes of  $f$ .

Find  $N + M$ .

1.

2.

3.

4.

5.

**QUESTION 34**

A function  $f$  has a Type II discontinuity at  $a$ .

Which of the following statements may be true?

I. The left-hand limit

exists.

II.  $f(a)$  exists.

III.  $f$  is bounded in some neighbourhood of  $a$ .

1. II only.
2. III only.
3. I and II only.
4. II and III only.
5. I, II, and III.