Математический анализ 1 — МИЭФ, 2025 demo midterm

МИЭФ | Математический анализ 1 | 2025 | demo midterm

Рисунки пока рендерятся в тестовом режиме и могут отличаться от исходных материалов.

QUESTION 1

The graph of $y = f(x)$ is shown below. Which of the alternatives **a–e** represents the graph of

$$y = f(|x|)?$$

Source graph of f and five candidate graphs for f(|x|)

Нужно проверить: "Coordinates are approximate because the source graph provides only tick marks, not an explicit formula."

1. a
2. b
3. c
4. d
5. e

QUESTION 2

Let

$$f(x) = 2x, \qquad g(x) = \frac{4}{x-1}.$$

The complete set of solutions of

$$f(g(x)) = g(f(x))$$

is:

1. $x_1 = \frac{1}{3}$.
2. $x_1 = 2$.
3. $x_1 = 3$.
4. $x_1 = -1,\ x_2 = 2$.
5. $x_1 = \frac{1}{3},\ x_2 = 2$.

QUESTION 3

Let

$$f(x) = x^5 - 1.$$

Then the inverse function f^{-1} is:

1. $$f^{-1}(x) = \frac{1}{\sqrt[5]{x} + 1}.$$

2. $$f^{-1}(x) = \frac{1}{\sqrt[5]{x + 1}}.$$

3. $$f^{-1}(x) = \sqrt[5]{x - 1}.$$

4. $$f^{-1}(x) = \sqrt[5]{x} - 1.$$

5. $$f^{-1}(x) = \sqrt[5]{x + 1}.$$

QUESTION 4

Find the principal period of

$$f(x) = 3 - 2\cos^2\left(\frac{\pi x}{3}\right).$$

1. $1.$
2. $2.$
3. $3.$
4. $5.$
5. $6.$

QUESTION 5

Let

$$f(x) = \cos(\arctan x).$$

What is the range of f?

1. $(0, 1).$
2. $(0, 1].$
3. $[0, 1].$
4. $(-1, 1).$
5. $[-1, 1].$

QUESTION 6

Let

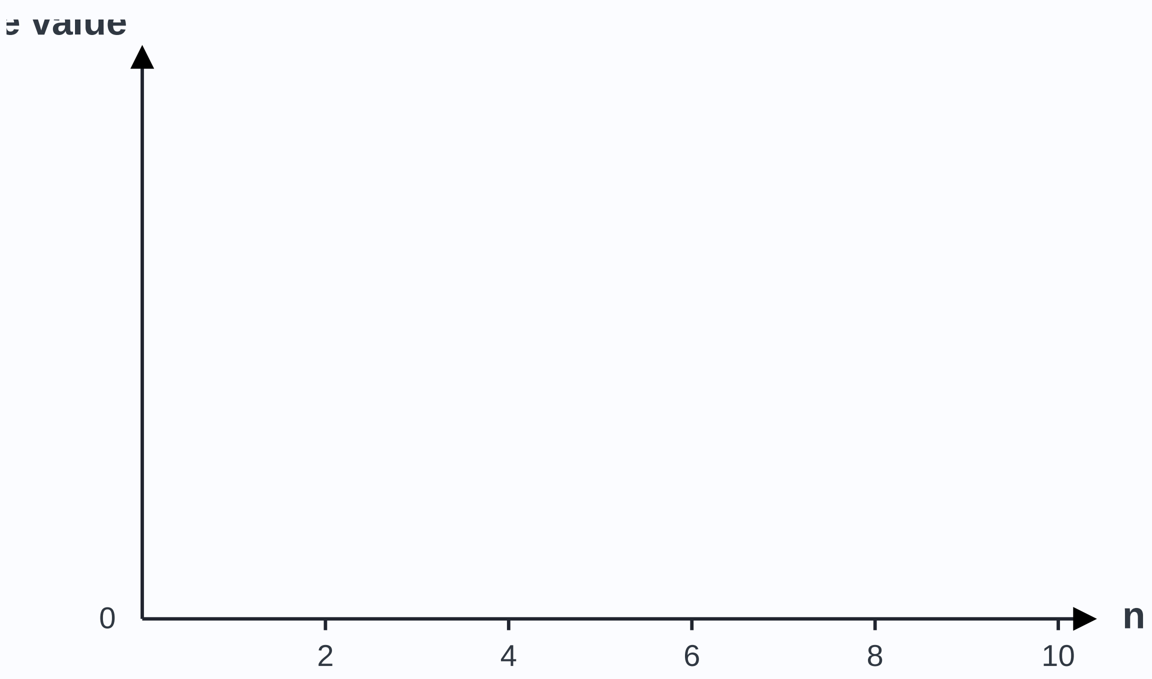
$$f(x) = \left| \sin x - \frac{1}{2} \right| .$$

The maximum value attained by f is:

1. at most $\frac{1}{2}$;
2. 1;
3. $\frac{3}{2}$;
4. $\frac{3}{4}$;
5. greater than $\frac{3}{2}$.

QUESTION 7

Match the five graphs with the five sequences.



Five discrete sequence graphs

Нужно проверить: "The plotted values are qualitative; exact numerical coordinates are not supplied."

Choose the correct matching:

1. $v_n = \dfrac{2a}{n+1},\quad w_n = \dfrac{an}{n+1},\quad x_n = \dfrac{a}{2^{n-1}},\quad y_n = \dfrac{(a-2)^n}{n!},\quad z_n = \dfrac{a-2}{\sqrt[n]{n}}.$

2. $v_n = \dfrac{a}{2^{n-1}},\quad w_n = \dfrac{(a-2)^n}{n!},\quad x_n = \dfrac{2a}{n+1},\quad y_n = \dfrac{an}{n+1},\quad z_n = \dfrac{a-2}{\sqrt[n]{n}}.$

3. $v_n = \dfrac{2a}{n+1},\quad w_n = \dfrac{an}{n+1},\quad x_n = \dfrac{a-2}{\sqrt[n]{n}},\quad y_n = \dfrac{(a-2)^n}{n!},\quad z_n = \dfrac{a}{2^{n-1}}.$

4. $v_n = \dfrac{(a-2)^n}{n!}, \quad w_n = \dfrac{a-2}{\sqrt[n]{n}}, \quad x_n = \dfrac{a}{2^{n-1}}, \quad y_n = \dfrac{2a}{n+1}, \quad z_n = \dfrac{an}{n+1}.$

5. $v_n = \dfrac{2a}{n+1}, \quad w_n = \dfrac{an}{n+1}, \quad x_n = \dfrac{(a-2)^n}{n!}, \quad y_n = \dfrac{a}{2^{n-1}}, \quad z_n = \dfrac{a-2}{\sqrt[n]{n}}.$

QUESTION 8

If

$$a_1 = 2, \qquad a_n = a_{n-1} + \frac{1}{3}$$

for every integer $n > 1$, then a_{101} equals:

1. $35 - \frac{2}{3}$.
2. $35 - \frac{1}{3}$.
3. 35.
4. $35 + \frac{1}{3}$.
5. $35 + \frac{2}{3}$.

QUESTION 9

Which of the following sequences converge?

$$\text{I. } a_n = \frac{5n}{2n-1}, \qquad \text{II. } c_n = \frac{e^n}{1+e^n}, \qquad \text{III. } b_n = \frac{e^n}{n}.$$

1. I only.
2. II only.
3. I and II only.
4. I and III only.
5. I, II, and III.

QUESTION 10

Which statements are true?

I. All bounded sequences are convergent.

II. If a sequence is unbounded, then it is infinitely large.

III. An unbounded sequence may have two limits.

1. I only.
2. II only.
3. II and III only.
4. I and III only.
5. None of the statements is true.

QUESTION 11

Find

$$\lim_{n\to\infty}\left(\frac{n^4+2n^2+4}{n^4-2n^2+4}\right)^{n^2}.$$

1. $1.$
2. $e.$
3. $e^2.$
4. $e^4.$
5. $+\infty.$

QUESTION 12

Let sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ be such that both difference sequences

$$\{x_n-y_n\} \qquad \text{and} \qquad \{y_n-z_n\}$$

are infinitesimally small.

Which of the following sequences must be convergent?

$$\text{I. } \{x_n-z_n\}, \qquad \text{II. } \{x_n+z_n\}, \qquad \text{III. } \left\{\frac{x_nz_n}{y_n^2}\right\}.$$

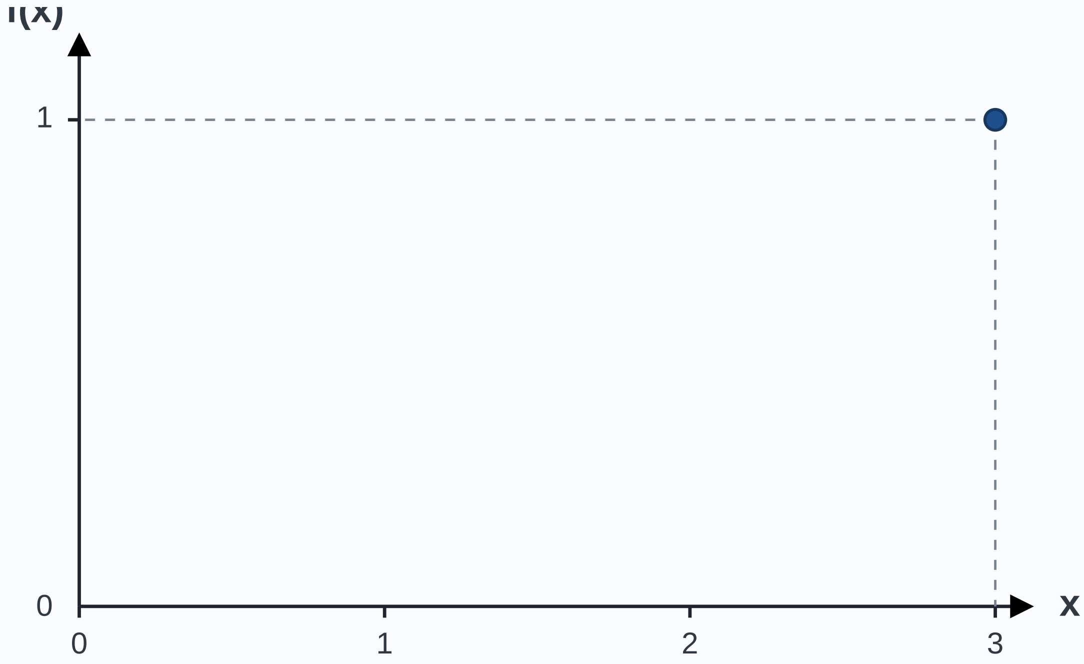
1. I only.
2. II only.
3. I and II only.
4. I and III only.
5. I, II, and III.

QUESTION 13

The graph of a function f is shown below.

For which values of c does

$$\lim_{x \to c} f(x) = 1?$$



Piecewise graph used to evaluate two-sided limits

Нужно проверить: "The exact height of the horizontal ray for x>3 is not labelled; only that it is below 1 matters."

1. 0 only.
2. 0 and 3 only.
3. -2 and 0 only.
4. -2 and 3 only.
5. -2, 0, and 3.

QUESTION 14

Find

$$\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x}.$$

1. The limit does not exist.
2. $-1.$
3. $0.$
4. $1.$
5. $2.$

QUESTION 15

Among the following choices of δ, which is the largest that can be used successfully for arbitrary $\varepsilon > 0$ in an ε–δ proof of

$$\lim_{x \to 2} (1 - 3x) = -5?$$

1. $\delta = 3\varepsilon.$
2. $\delta = \varepsilon.$
3. $\delta = \frac{1}{2}\varepsilon.$
4. $\delta = \frac{1}{4}\varepsilon.$
5. $\delta = \frac{1}{5}\varepsilon.$

QUESTION 16

Let

$$f(x) = \sin\left(\frac{x+1}{x^2}\right).$$

Which statements are true?

I. The graph of f has a horizontal asymptote $y = 0$.

II. The graph of f has a horizontal asymptote $y = 1$.

III. The graph of f has a vertical asymptote at $x = 0$.

1. I only.
2. II only.
3. III only.
4. I and III only.
5. II and III only.

QUESTION 17

Find the asymptote of

$$y = \frac{x^3}{\sqrt{x^4 + x^2 + 1}}$$

as $x \to -\infty$.

1. $y = 1.$
2. $y = x.$
3. $y = -x.$
4. $y = x + 1.$
5. $y = -x + 1.$

QUESTION 18

Let

$$f(x) = \frac{x^3}{x^2 - 4} + |x|.$$

The graph of $y = f(x)$ has:

1. two vertical asymptotes, one horizontal asymptote, and one slant asymptote with nonzero slope;
2. two vertical asymptotes and two slant asymptotes with slopes of different signs;
3. one vertical asymptote and two slant asymptotes with positive slopes;
4. one vertical asymptote and three slant asymptotes with positive slopes;
5. three different asymptotes.

QUESTION 19

Find

$$\lim_{x\to+\infty} \frac{\ln\left(1+\sqrt{x}+\sqrt[3]{x}\right)}{\ln\left(1+\sqrt[3]{x}+\sqrt[4]{x}\right)}.$$

1. $1.$
2. $0.$
3. $\frac{2}{3}.$
4. $\frac{3}{4}.$
5. $\frac{3}{2}.$

QUESTION 20

Let f be continuous at $x = 2$, and

$$f(x) = \begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \neq 2, \\ k, & x = 2. \end{cases}$$

Then:

1. $k = 0.$
2. $k = \frac{1}{6}.$
3. $k = \frac{1}{3}.$
4. $k = 1.$
5. $k = \frac{7}{5}.$

QUESTION 21

Let f be a continuous function defined by

$$f(x) = \sqrt{\tan^2 x - 1}.$$

Which interval could be the domain of f?

1. $\left(\frac{3\pi}{4}, \pi\right)$.
2. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.
3. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$.
4. $\left(-\frac{\pi}{4}, 0\right)$.
5. $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$.

QUESTION 22

Let f be continuous on the closed interval $[-3, 6]$. If

$$f(-3) = -1, \qquad f(6) = 3,$$

then the Intermediate Value Theorem guarantees that:

1. $f(0) = 0$;
2. $f(x) \leq f(c)$ for some $c \in [-3, 6]$;
3. $-1 \leq f(x) \leq 3$ for every x between -3 and 6;
4. $f(c) = 0$ for at least one c between -1 and 3;
5. $f(c) = 1$ for at least one c between -3 and 6.

QUESTION 23

According to the Intermediate Value Theorem, which statements are true?

I. The equation

$$x^{20} + x^{23} - 2022 = 0$$

has at least one root in $[-10, 10]$.

II. The function

$$f(x) = \log_2 x$$

takes the value 11 in $[2022, 2202]$.

III. The function

$$f(x) = \frac{1}{x^2 - 6x + 8}$$

takes the value 0 in $[1, 3]$.

1. III only.
2. I and III only.
3. II and III only.
4. I and II only.
5. I, II, and III.

QUESTION 24

Let $a, b, c, d \in \mathbb{R}$. The polynomial equation

$$x^7 + ax^5 + bx^3 + cx + d = 0$$

must have:

1. only one root;
2. at least one root;
3. an even number of roots;
4. no negative roots;
5. no positive roots.

QUESTION 25

At how many points do the graphs

$$y = x^{12} \qquad \text{and} \qquad y = 2^x$$

intersect?

1. None.
2. One.
3. Two.
4. Three.
5. Four.

QUESTION 26

Let k be the number of real solutions of

$$e^x + x - 2 = 0$$

in $[0, 1]$, and let n be the number of real solutions outside $[0, 1]$.

Which statement is true?

1. $k = 0$ and $n = 1$.
2. $k = 1$ and $n = 0$.
3. $k = n = 1$.
4. $k > 1$.
5. $n > 1$.

QUESTION 27

Let

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2} - x}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Which statement is true?

1. f is continuous for every x.
2. f has a removable discontinuity at $x = 0$.
3. f has a jump discontinuity at $x = 0$, and

$$\lim_{x \to 0^+} f(x) = f(0).$$

4. f has a jump discontinuity at $x = 0$, and

$$\lim_{x \to 0^-} f(x) = f(0).$$

5. f has a Type II discontinuity at $x = 0$.

QUESTION 28

Consider

$$f(x) = \begin{cases} 2^{1/x}, & x < 0, \\ 1, & 0 \le x \le 1, \\ \ln(x^2 + x - 2), & x > 1. \end{cases}$$

The function f has:

1. only one jump discontinuity;
2. two jump discontinuities;
3. one Type II discontinuity only;
4. one jump discontinuity and one Type II discontinuity;
5. one jump discontinuity and one removable discontinuity.

QUESTION 29

Consider

$$f(x) = \arctan\left(\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2}\right).$$

Let D be the number of Type II discontinuities of f, and let A be the number of distinct asymptotes of f.

Find $D + A$.

1. $1.$
2. $3.$
3. $6.$
4. $2.$
5. $8.$

QUESTION 30

A function f has a Type II discontinuity at $x = a$.

Which of the following statements may be true?

I. The left-hand limit

$$\lim_{x \to a^-} f(x)$$

exists.

II. $f(a)$ exists.

III. f is bounded in some neighbourhood of a.

1. II only.
2. III only.
3. I and II only.
4. II and III only.
5. I, II, and III.

QUESTION 31

Open Question 1

Original label in the source: **Question 1**.

Let f be the function defined by

$$f(x) = e^{\sqrt{2x^2-1}}.$$

(a) Is $f(x)$ even, odd, or neither? Justify your answer.

(b) Find the domain of $f(x)$. Justify your answer.

(c) Find the range of $f(x)$. Justify your answer.

(d) Find all points x where $f(x)$ is continuous. Justify your answer.

(e) Find

$$\lim_{h\to 0} \frac{\ln(f(1+h)) - \ln(f(1))}{h}.$$

QUESTION 32

Open Question 2

Original label in the source: **Question 2**.

Let f be the function defined by

$$f(x) = \frac{2x-2}{x^2+x-2}.$$

(a) For what values of x is $f(x)$ discontinuous?

(b) For each point of discontinuity a found in part (a), find

$$\lim_{x \to a} f(x),$$

or justify that the limit does not exist.

(c) Find equations of all asymptotes, slant or vertical, to the graph of f. Justify your answer.

(d) A rational function

$$g(x) = \frac{b}{c+x}$$

satisfies

$$f(x) = g(x)$$

at all points

$$x \in D[f].$$

Find b and c.

QUESTION 33

Open Question 3

Original label in the source: **Question 3**.

Consider the sequence

$$a_n = \frac{2^n}{n!}, \qquad n = 1, 2, 3, \dots$$

(a) Classify the sequence $\{a_n\}$ as increasing, nondecreasing, nonincreasing, decreasing, or neither. Justify your answer.

(b) Is the sequence $\{a_n\}$ bounded? Justify your answer.

(c) What conclusion can you draw from parts (a) and (b)? Mention the names of all theorems you use.

(d) Let $\{x_n\}$ be a convergent sequence:

$$\lim_{n\to\infty} x_n = L.$$

Find

$$R = \lim_{n\to\infty} \frac{x_{n+1}}{x_n}.$$

(e) Calculate

$$A = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

if

$$a_n = \frac{2^n}{n!}.$$

Are your answers for A and R from part (d) consistent?

QUESTION 34

Open Question 4

Original label in the source: **Question 4**.

Let f be bounded and continuous on the interval

$$[0, \infty),$$

and suppose

$$f(x) \neq 0$$

for every

$$x \geq 0.$$

Which of the following statements **must be true**?

(a) There exists a point

$$c \in [0, \infty)$$

such that

$$f(x) \leq f(c)$$

for every

$$x \geq 0.$$

(b) The function f does not change sign on the interval

$$[0, \infty).$$

(c) There exists a finite number L such that

$$\lim_{x \to +\infty} f(x) = L.$$

If you think a statement is always true, explain why, mentioning the relevant theorems.

If you think a statement is false, provide a counterexample.