

Теория вероятностей и математическая статистика — МИЭФ, 2025 demo midterm

МИЭФ

Теория вероятностей и математическая статистика

2025

demo midterm

Рисунки пока рендерятся в тестовом режиме и могут отличаться от исходных материалов.

PROBLEM 1**8% of total score**

(a) The distribution of random variable X is given by the table:

X	-2	-1	0	1
P	0.3	α	0.2	0.35

Find α .

(b) A die is rolled. If the number on the die is less than 3, we buy shares; otherwise, we place money in a bank account.

Find the probability that the die shows 5 given that we place money in the bank account.

(c) The probability that a day is sunny is 0.5, the probability that it is snowy is 0.3, and the probability that it is both sunny and snowy is 0.2.

Find the probability that the day is neither sunny nor snowy.

PROBLEM 2

8% of total score

(a) A die is rolled.

- If it shows an even number, that number is recorded.
- If it shows an odd number, the die is rolled again, and the result of the second roll is recorded.

Find the probability that the recorded number is 6.

(b) In the experiment from part (a), suppose the recorded number is 6.

Find the conditional probability that the die was rolled twice.

(c) A team plays two games. It wins the first game with probability $\frac{1}{2}$ and the second game with probability $\frac{1}{3}$. The games are independent.

Find the probability that the team wins exactly one of the two games.

PROBLEM 3

8% of total score

(a) A box contains 5 white balls, 2 black balls, and 1 red ball.

Balls are drawn one at a time **with replacement** until a white or red ball is drawn.

Let X be the number of draws, including the final draw on which a white or red ball appears.

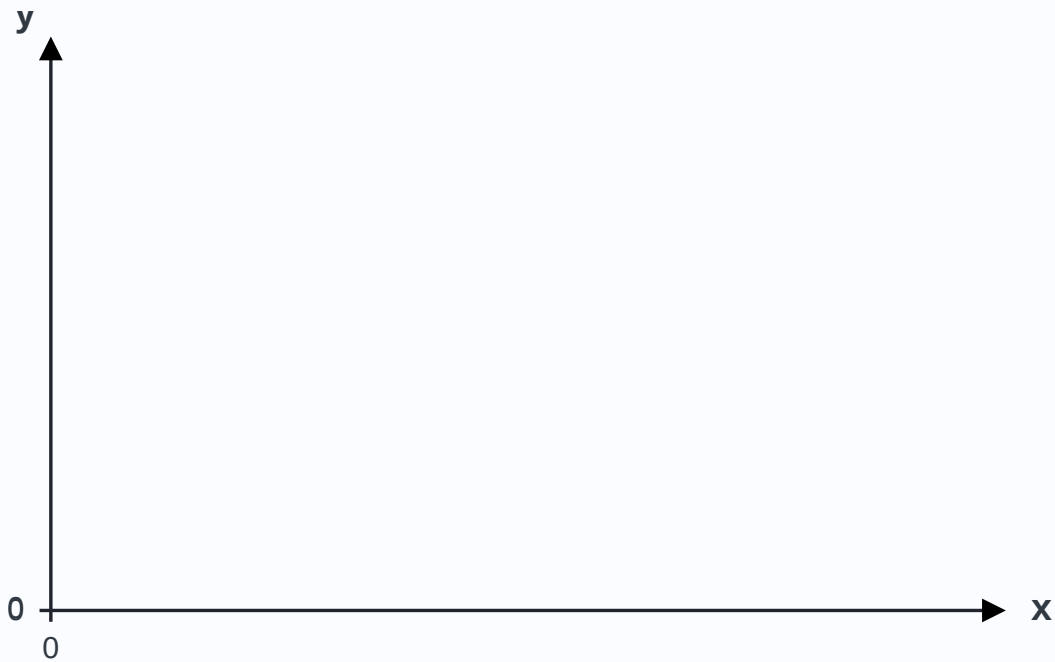
Find

$$\mathbb{E}(X).$$

(b) On the following diagram, show the event

$$C \setminus (A \cup B)$$

by hatching the corresponding region.



Three-set Venn diagram for C minus the union of A and B

(c) The distribution of random variable X is given by the table:

X	-1	1	2
P	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Find

$$\text{Var}(3 \cdot 2^X - 5).$$

PROBLEM 4

8% of total score

(a) A random variable X has a geometric distribution with parameter p :

$$X \sim \text{Geom}(p).$$

Find

$$\mathbb{E}\left(\frac{1}{2^X}\right)$$

as a function of p .

The answer must be written as a formula. An answer in the form of a series is not accepted.

(b) There are 4 courses in economics, 3 courses in mathematics, and 2 courses in computer science. A student must choose 5 courses.

Find the probability that the student chooses:

- 2 courses in economics;
- 2 courses in mathematics;
- 1 course in computer science.

(c) Find the binomial coefficient

$$\binom{11}{4}.$$

PROBLEM 5

8% of total score

A computer program generates random 4-digit PIN codes. Every combination of digits from 0 to 9 is possible, and all combinations have the same probability of being generated. Digits may repeat.

(a) How many different 4-digit PIN codes can be generated?

(b) Suppose the program randomly generates 100 PIN codes, independently of one another.

Find the probability that all generated PIN codes are different.

(c) Suppose 100 PIN codes are generated independently, as in part (b).

Find the expected number of distinct PIN codes among them.

PROBLEM 6

20% of total score

An individual invests in stocks, bonds, and exchange-traded funds (ETFs).

The investor chooses among:

- 5 types of stocks;
- 4 types of bonds;
- 3 types of ETFs.

On every working day, Monday through Friday, the investor randomly selects one of the 12 financial assets and buys one unit. The same asset may be chosen on different days. The investor does not trade on weekends.

(a) Describe the classical probability space for the investor's choices during one working week.

Define an elementary outcome and the probability of each elementary outcome.

Find the probability that the investor chooses only stocks and bonds during the week.

(b) Let X be the number of working days during the week on which the investor chooses a stock or a bond.

Find:

- the distribution of X ;
- $\mathbb{E}(X)$;
- the standard deviation of X .

Express the probability found in part (a) in terms of X .

(c) Find the expected number of different asset classes selected by the investor during the week.

For example:

- if the investor chooses only stocks during the week, the number of asset classes is 1;

- if the investor chooses bonds and ETFs, the number is 2;
- if the investor chooses stocks, bonds, and ETFs, the number is 3.

PROBLEM 7

20% of total score

The objective of a popular computer game is to defeat the final boss.

At the beginning of the game, a player chooses the difficulty level:

- 50% of players choose **easy**;
- 40% choose **medium**;
- 10% choose **hard**.

Conditional on the chosen difficulty:

- on easy, the player defeats the boss with probability 0.5;
- on medium, with probability 0.25;
- on hard, with probability 0.05.

A player who defeats the boss receives:

- 25 points on easy;
- 50 points on medium;
- 150 points on hard.

(a) Find the probability that a randomly selected player defeats the final boss.

(b) Find the probability that a player defeats the final boss given that the player did not choose easy difficulty.

(c) Two players independently attempt to defeat the final boss. It is known that the first player chose medium difficulty, while the difficulty chosen by the second player is unknown.

Find the most likely **positive** total number of points earned by the two players, excluding the case in which both players lose.

State this most likely total and the probability of obtaining it.

PROBLEM 8

20% of total score

Two players, A and B , play a game consisting of independent rounds.

Initially:

A has 40 dollars, B has 60 dollars.

Each round results in a win for one player and a loss for the other; draws are impossible.

- If player A wins, player B pays player A 2 dollars.
- If player B wins, player A pays player B 1 dollar.

Let p be the probability that player A wins a round.

(a) Let X be player A 's profit in one round, that is, the amount won or lost by player A .

Find the distribution, expectation, and variance of X as functions of p .

(b) Find the value of p for which the game is fair, meaning that the expected profits of players A and B in any round are equal.

Let V_n and W_n denote the amounts of money held by players A and B , respectively, after n rounds.

For the fair game, find

$$\mathbb{E}(V_n) \quad \text{and} \quad \mathbb{E}(W_n).$$

(c) Let

$$p = \frac{1}{2}.$$

Find the probability that after 8 rounds player A has more money than player B .