Теория вероятностей и математическая статистика — МИЭФ, 2026 demo midterm

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Рисунки пока рендерятся в тестовом режиме и могут отличаться от исходных материалов.

PROBLEM 1

8% of total score

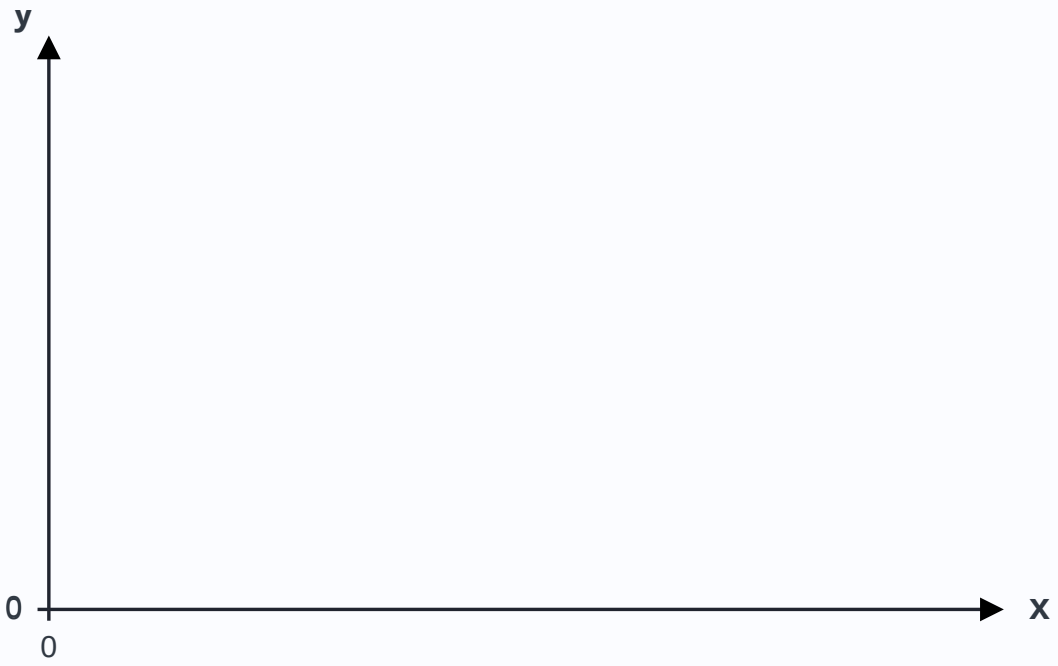
(a) Two dice are rolled.

Find the probability that the dice show the same number of points.

(b) On the following diagram, show the event

$$(A \cup B) \cap C$$

by hatching the corresponding region.



Three-set Venn diagram for the intersection of C with A union B

(c) How many different words can be formed by rearranging the letters in the word **STAT**?

A word may be any sequence of the letters and need not have a meaning. Include the original word **STAT** in the count.

PROBLEM 2

8% of total score

(a) A random variable X has a binomial distribution with parameters

$$n = 10, \qquad p = 0.4,$$

that is,

$$X \sim \text{Binom}(10, 0.4).$$

Find

$$\mathbb{E}(X^2).$$

(b) Let

$$P(A) = 0.2, \qquad P(B) = 0.4, \qquad P(A \cap B) = 0.05.$$

Find

$$P(A \cup B).$$

(c) Consider passwords of length 5 consisting of **distinct digits**.

Find the number of passwords for which the second digit is less than the fourth digit.

PROBLEM 3

8% of total score

(a) Five fair coins are tossed.

Find the probability that at least two coins land heads up.

(b) Find the expected value of random variable X with the following distribution:

X	-5	0	5	10
P	0.1	0.2	0.3	0.4

(c) There are three empty boxes and three balls. Each ball is independently and uniformly placed into one of the boxes. More than one ball may be placed in the same box.

Let X be the number of non-empty boxes after all three balls have been placed.

Find the distribution of X.

PROBLEM 4

8% of total score

(a) A random variable X has a geometric distribution with parameter p:

$$X \sim \mathrm{Geom}(p).$$

Find

$$\mathbb{E}\left(\frac{1}{2^X}\right)$$

as a function of p.

The answer must be written as a formula. An answer in the form of a series is not accepted.

(b) There are 4 courses in economics, 3 courses in mathematics, and 2 courses in computer science. A student must choose 5 courses.

Find the probability that the student chooses:

- 2 courses in economics;
- 2 courses in mathematics;
- 1 course in computer science.

(c) Find the binomial coefficient

$$\binom{11}{4}.$$

PROBLEM 5

8% of total score

A computer program generates random 4-digit PIN codes. Every combination of digits from 0 to 9 is possible, and all combinations have the same probability of being generated. Digits may repeat.

(a) How many different 4-digit PIN codes can be generated?

(b) Suppose the program randomly generates 100 PIN codes, independently of one another.

Find the probability that all generated PIN codes are different.

(c) Suppose 100 PIN codes are generated independently, as in part (b).

Find the expected number of distinct PIN codes among them.

PROBLEM 6

20% of total score

An economist forecasts the country's GDP growth for the next year.

There are two scenarios:

- an optimistic scenario, which occurs with probability 60%;
- a pessimistic scenario, which occurs with probability 40%.

Conditional GDP-growth distributions are given below.

Optimistic scenario:

GDP growth	0%	3%	5%
Probability	20%	30%	50%

Pessimistic scenario:

GDP growth	-3%	0%	3%
Probability	50%	30%	20%

(a) Find the probability that GDP will fall next year according to the forecast.

(b) Let X denote GDP growth next year. GDP growth may be negative.

Find the distribution of X.

(c) Find

$$\mathbb{E}(X).$$

PROBLEM 7

20% of total score

The objective of a popular computer game is to defeat the final boss.

At the beginning of the game, a player chooses the difficulty level:

- 50% of players choose **easy**;
- 40% choose **medium**;
- 10% choose **hard**.

Conditional on the chosen difficulty:

- on easy, the player defeats the boss with probability 0.5;
- on medium, with probability 0.25;
- on hard, with probability 0.05.

A player who defeats the boss receives:

- 25 points on easy;
- 50 points on medium;
- 150 points on hard.

(a) Find the probability that a randomly selected player defeats the final boss.

(b) Find the probability that a player defeats the final boss given that the player did not choose easy difficulty.

(c) Two players independently attempt to defeat the final boss. It is known that the first player chose medium difficulty, while the difficulty chosen by the second player is unknown.

Find the most likely **positive** total number of points earned by the two players, excluding the case in which both players lose.

State this most likely total and the probability of obtaining it.

PROBLEM 8

20% of total score

Two players, A and B, play a game consisting of independent rounds.

Initially:

$$A \text{ has } 40 \text{ dollars}, \qquad B \text{ has } 60 \text{ dollars}.$$

Each round results in a win for one player and a loss for the other; draws are impossible.

- If player A wins, player B pays player A 2 dollars.
- If player B wins, player A pays player B 1 dollar.

Let p be the probability that player A wins a round.

(a) Let X be player A's profit in one round, that is, the amount won or lost by player A.

Find the distribution, expectation, and variance of X as functions of p.

(b) Find the value of p for which the game is fair, meaning that the expected profits of players A and B in any round are equal.

Let V_n and W_n denote the amounts of money held by players A and B, respectively, after n rounds.

For the fair game, find

$$\mathbb{E}(V_n) \qquad \text{and} \qquad \mathbb{E}(W_n).$$

(c) Let

$$p = \frac{1}{2}.$$

Find the probability that after 8 rounds player A has more money than player B.