@zachemimenno Архив экзаменов прошлых лет

Микроэкономика 2 — Совбак ВШЭ и РЭШ, 2026 final

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PROBLEM 1

Repeated Bertrand competition — 25 points

Consider a market with two identical firms operating in a homogeneous-product market with demand

$$Q(p) = 100 - p$$

and constant marginal cost equal to 10. Firms compete by simultaneously setting prices, that is, they play a Bertrand game.

1 (3 points) In a static environment, what is the unique Nash-equilibrium price of this game? What are the profits of each firm?

2 (5 points) Suppose the firms could legally sign a cartel agreement. What price would they optimally set if they shared the market equally? What would each firm's profit be?

Why would this outcome be impossible if the firms were not allowed to communicate and write such a contract?

3 (7 points) Suppose the two firms interact repeatedly for infinitely many periods.

Construct an equilibrium that allows the firms to sustain the cartel price indefinitely using a grim-trigger strategy. Write down the strategy in detail.

What is the smallest discount factor δ that sustains tacit collusion as a subgame-perfect Nash-equilibrium outcome?

4 (10 points) Suppose market demand changes over time at growth rate μ and demand in period t is

$$D(p_t) = \mu^t(100 - p_t),$$

where

$$\mu\delta < 1.$$

The growth rate may be greater than one, corresponding to a growing market, or less than one, corresponding to a declining market.

Derive the set of discount factors for which maximal collusion can be sustained as an equilibrium of the infinitely repeated game. Use the same grim-trigger strategy as in part 3.

What does the model predict about the relative ease of sustaining collusion in expanding and declining industries?

PROBLEM 2

Compatibility choice in a Hotelling market — 50 points

Consumers are uniformly distributed on the unit interval

$$x \in [0, 1].$$

There are two firms, A and B, located at the endpoints:

$$l_A = 0, \qquad l_B = 1.$$

Each firm produces at constant marginal cost $c \geq 0$. Each consumer buys at most one unit. The utility of a consumer located at x from buying from firm $i \in \{A, B\}$ is

$$u_i(x) = v - p_i - t|x - l_i| - \sigma s_i,$$

where:

- p_i is the price charged by firm i;
- $t > 0$ is the transportation, or horizontal-differentiation, parameter;
- $\sigma > 0$ is the compatibility-disutility parameter.

Each firm chooses whether its product is compatible with an industry standard:

- if firm i is compatible, then $s_i = 0$;
- if firm i is incompatible, then $s_i = 1$.

Thus incompatibility adds disutility σ. Compatibility is costly: a compatible firm incurs a fixed cost $F > 0$, while an incompatible firm incurs no fixed cost. The compatibility cost of firm i is therefore

$$(1 - s_i)F.$$

The game has three stages:

1. Firms simultaneously choose whether to be compatible or incompatible.
2. Firms observe the compatibility choices and simultaneously choose prices p_A and p_B.
3. Consumers choose which firm to buy from.

Assume v is sufficiently large for the market to be fully covered in equilibrium.

1 (5 points) Fix prices p_A, p_B and compatibility choices (s_A, s_B).

Derive the firms' demands D_A and D_B. Assume prices and compatibility choices are such that each firm serves some consumers.

2 (8 points) Fix compatibility choices (s_A, s_B).

Solve for the Nash-equilibrium prices p_A^* and p_B^*, assuming each firm serves some consumers in equilibrium.

Show that

$$\sigma < 3t$$

ensures that each firm serves some consumers in equilibrium for every possible compatibility profile (s_A, s_B).

Maintain the assumption $\sigma < 3t$ for the remainder of the problem.

3 (12 points) For fixed compatibility choices (s_A, s_B) and the equilibrium prices from part 2, calculate:

- the payoff of each firm;
- consumer welfare;
- social welfare, defined as the sum of firms' payoffs and consumer welfare.

Does compatibility benefit consumers?

4 (15 points) Characterize all pure-strategy equilibria of the compatibility game.

Find the equilibrium compatibility choices

$$s_A^*, \qquad s_B^*,$$

and the corresponding prices and demands.

5 (10 points) Calculate the socially efficient compatibility choices

$$s_A^{**}, \qquad s_B^{**},$$

that maximize the social welfare derived in part 3.

Compare the socially efficient choices with the equilibrium choices from part 4.

Do firms choose too much or too little compatibility relative to the social optimum? Provide intuition.

PROBLEM 3

Adverse selection and warranties in a used-car market — 25 points

There is one seller and one buyer. The seller owns one used car, and the buyer wants to buy at most one car.

Cars are of two types:

- a **peach**, or good car, with probability $\alpha \in (0, 1)$;
- a **lemon**, or bad car, with probability $1 - \alpha$.

The seller privately observes the type of her car; the buyer does not.

Valuations are:

$$\text{Buyer:} \quad \bar{v}_b \text{ for a peach,} \qquad v_b \text{ for a lemon,}$$

with

$$\bar{v}_b > v_b > 0,$$

and

$$\text{Seller:} \quad \bar{v}_s \text{ for a peach,} \qquad v_s \text{ for a lemon,}$$

with

$$\bar{v}_s > v_s > 0.$$

Assume trade is efficient for both types:

$$\bar{v}_b > \bar{v}_s, \qquad v_b > v_s,$$

and additionally

$$v_b < \bar{v}_s.$$

All bargaining power lies on the seller's side. The market price therefore equals the buyer's expected valuation conditional on the information available to the buyer, so the buyer obtains zero expected utility.

1 (7 points) Suppose the seller cannot take any action other than deciding whether to sell at the market price.

(a) (4 points) Consider a candidate equilibrium in which both types of cars are sold.

What is the market price p_{pool}? Derive the condition on α under which both seller types want to sell at this price.

(b) (3 points) If the condition from part (a) fails, show that there is an equilibrium in which only lemons are sold.

Find the price p_{lem} and explain why the peach seller does not sell.

2 (12 points) Suppose that, before selling, the seller can publicly choose whether to offer a transferable warranty

$$W \in \{0, 1\}.$$

If $W = 1$ and the car is sold, the seller bears an expected warranty cost k_P if the car is a peach and k_L if the car is a lemon, where

$$0 \leq k_P < k_L.$$

The warranty cost does not improve the car's quality and is completely wasteful.

If $W = 0$, there is no warranty cost.

The buyer observes W.

Consider a separating equilibrium in which the peach seller chooses $W = 1$ and the lemon seller chooses $W = 0$.

What are the market prices p_1 for cars with $W = 1$ and p_0 for cars with $W = 0$?

Derive the conditions on the parameters under which this separating equilibrium exists.

3 (6 points) Let

$$\bar{v}_b = 10, \qquad v_b = 4, \qquad \bar{v}_s = 8, \qquad v_s = 2,$$

$$\alpha = 0.3, \qquad k_P = 1, \qquad k_L = 7.$$

Find the equilibrium outcome in the benchmark model from part 1: determine which types trade and at what price.

Check whether the separating warranty equilibrium from part 2 exists. If it does, state which types trade and at what prices.

Compute and compare expected total surplus in the two settings.